

# Does Electoral Volatility Beget Strong Alliances? A Theory of Multi-Party Competition

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## Abstract

In multi-party systems parties often form alliances before elections. Despite being pervasive, little is known about the conditions facilitating different configurations of pre-electoral alliances. This paper presents a model of electoral competition in which parties can form alliances before elections, and decide how binding these should be. Parties face a dynamic trade-off between insuring themselves against large shifts in public opinion and allowing flexibility to respond to future changes in voters' preferences. The model shows that more binding alliances such as mergers emerge in equilibrium when electoral volatility is high; otherwise, parties form more flexible pre-electoral coalitions. It also suggests that some power concentration is needed for alliances to emerge in equilibrium, whereas parties run alone under consensual democracies that share power among all parties. These results have implications for the process of party system stabilization.

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# 1. Introduction

In multi-party systems, future alternative governments are often offered to voters by different configurations of pre-electoral alliances between political parties. A common way for different parties to form an alliance in a given election is to support joint candidates, while keeping their separate identities. This form of alliance, typically referred to as a pre-electoral coalition, is often chosen by parties to join forces against strong opponent candidates. For example, recent evidence from Mexican and Finnish local elections demonstrates that parties are willing to form pre-electoral alliances to remove entrenched incumbent parties from office (Frey, López-Moctezuma and Montero, 2021; Hortala-Vallve, Meriläinen and Tukiainen, 2021).

Alternatively, parties can join forces by merging into new political entities. Mergers are a common alternative to pre-electoral coalitions. In Europe, for example, mergers have occurred on average every third electoral period since World War II.<sup>1</sup> Furthermore, political leaders consider the option of merging even more frequently than what the number of occurrences suggests. For example, in the UK, mergers are an often discussed option, as indicated by frequent media reports about the advantages of a merger between the UK Liberal Democratic Party and the Labour Party.<sup>2</sup>

Mergers lead to significant changes in the party system. The Italian political landscape completely changed in 2007, when mergers across the ideological spectrum effectively transformed the system into bipolarism, with two main competing electoral cartels.<sup>3</sup> Pre-electoral alliances such as mergers facilitate the formation of durable parties and can reduce party system fragmentation. For instance, the fusion leading to the formation of the Christian Democratic Appeal in the Netherlands in 1980 helped to eliminate the cleavage between Catholics

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<sup>1</sup>Ibenskas (2016) collected a dataset that considers 280 democratic elections in the postwar period in European countries. Overall, the dataset includes 94 mergers formed by 216 parties. These mergers occurred over 59 electoral periods and were predominantly formed by two parties.

<sup>2</sup>Cf. Jenkins, S. (2019) 'The Lib Dems helped the Tories to victory again. Now they should disband', The Guardian, 16 December: <https://www.theguardian.com/commentisfree/2019/dec/16/lib-dems-tories-split-vote-labour>

<sup>3</sup>The first fusion occurred between April and October 2007, when the *Democratici di Sinistra* — the largest of the successor parties of the former *Partito Comunista Italiano* — merged with *La Margherita* to form the Democratic Party (PD). A few months later, Berlusconi's *Forza Italia* merged with the right-wing *Alleanza Nazionale* to form the *Popolo della Libertà* in November. Triggered by the creation of the PD, a smaller merger occurred that same year between the parties of the radical left, which merged under the name of *Sinistra Arcobaleno*.

and Protestants in the Dutch party system and substantially reduced party system fragmentation (Koole, 1994). Outside Europe, the merger between the Progressive-Conservative (PC) and the Canadian Alliance parties in 2003 created a new right-wing formation, significantly altering the Canadian party system and subsequent voting behavior (Bélanger and Godbout, 2010).

Despite the evidence showing that parties across the world are increasingly seen to join forces before election (Powell Jr, 2000; Golder, 2006) — adopting various governance configurations — pre-electoral alliances have not received much attention from the literature, which overwhelmingly focuses on government (post-electoral) coalitions. However, understanding the incentives behind different configurations of pre-electoral alliances is crucial, as these can have significant consequences on election outcomes, government composition, policies and the development of party systems. This paper proposes a model of elections in which parties can form pre-electoral coalitions and mergers before elections, and studies which features of the electoral environment facilitate the formation of each type of alliance.

What are the defining features that distinguish mergers from pre-electoral coalitions (hereafter, PECs)? The first dimension of variation is the *scope* of parties' cooperation (Ibenskas and Bolleyer, 2018). Golder (2006, 28) defines a PEC as a “collection of parties that do not compete independently in an election, either because they publicly agree to coordinate their campaigns, run joint candidates or joint lists, or enter government together following the election.” Parties belonging to a coalition cooperate in specific areas (e.g., electoral competition through the formation of joint lists of candidates), while still competing with their separate identities in other areas (e.g., member recruitment). In contrast, mergers are defined as “the amalgamation of two or more independent parties into a single party organization” (Ibenskas, 2016). This complete fusion implies that cooperation becomes unrestricted: a merger entails an agreement to become a new organization, which presupposes unrestricted and universal cooperation among the constituent parties.

The literature provides an intuitive analysis of the factors that should facilitate the formation of mergers. On the one hand, mergers are less likely to form among highly ideologically distant parties and when parties have established identities. On the other, a highly disproport-

tional electoral system encourages parties to merge to improve their post-electoral legislative weight. However, very similar incentives drive parties' choice to join PECs, without relinquishing their own identity or party brand. When, and why, do parties retain their separate identities rather than merge into a larger party?

I argue that parties' choice over different forms of pre-electoral alliances crucially depends on *electoral volatility*, reflecting the extent to which voters' preferences change between subsequent elections. Electoral volatility can be thought as being inversely related to partisanship: if voters are highly partisan, voters' preferences are likely to stay constant over time. Parties face a dynamic trade-off: while mergers insure constituent parties against unfavorable shifts in the electorate's preferences, these binding forms of alliances come at the cost of losing the opportunity to join more advantageous coalitions in the future. Conversely, alliances that allow parties to maintain their identity offer more flexibility to respond to changes in voters' preferences.

To analyze this trade-off, the paper introduces a model of multi-party electoral competition where policy-motivated parties can form alliances before elections. In the model, each party is associated with a different policy platform, or "brand." While these brands are fixed, parties can change the policy platform that voters evaluate by joining pre-electoral alliances. In particular, the platform resulting from an alliance is a convex combination of the constituent parties' platforms. Besides competing alone and forming PECs — whereby distinct parties run with a common platform — parties can constitute new political entities by merging. A merger is a binding arrangement that solidifies the relative power constituent parties have at a given point in time. Conversely, PECs preserve parties' identities, allowing parties to be more flexible to changes in the electoral environment.

The model features a two-period game between three parties. In each period, parties can form mergers or PECs (or run alone), and an election takes place.<sup>4</sup> While mergers persist in the future election, PECs are only temporary alliances that need to be renegotiated in each period. This assumption reflects the empirical regularity that PECs are often revisited: indeed, coalition candidates' lists are typically renegotiated before each election. In contrast, once a

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<sup>4</sup>More precisely, I assume that in each period the centrist party can propose either type of alliance to the left and right parties. This assumption rules out alliances between the left and the right.

merger is formed there is a high cost for terminating it, and empirical evidence suggests that mergers persist more easily across elections, as shown in Table 1.

**Number and Stability of Electoral Coalitions and Mergers per Country**

Country	Average Number of Unstable Coalitions per Election	Average Number of Stable Coalitions per Election	Average Number of Unstable Mergers per Electoral Period	Average Number of Stable Mergers per Electoral Period
Bulgaria	1.5	0.7	0.2	0.3
Czech Republic	0.5	0.5	0.0	0.7
Estonia	1.0	0.8	0.3	1.3
Hungary	0.3	0.5	0.0	0.0
Latvia	0.7	1.5	0.2	1.0
Lithuania	1.0	0.2	0.3	1.2
Poland	1.8	0.5	0.2	0.8
Romania	1.0	0.3	0.0	1.2
Slovakia	0.7	1.0	0.0	1.0
Slovenia	0.2	0.0	0.2	0.7
Total	1.0	0.6	0.1	0.9

**Table 1** – Number of stable/unstable coalitions and mergers in the first six electoral periods in 10 countries in Central and Eastern Europe. Source: [Ibenskas and Bolleyer \(2018\)](#).

When deciding which type of alliance to choose, if any, the main trade-off parties face is between the flexibility provided by a PEC and the insurance against large shifts in public opinion that a merger guarantees. Keeping separate identities allows parties to respond to changes in voters’ preferences, which are modeled as a move of Nature in favor of either party that takes place between elections. By merging, parties commit to the relative power held at the time of the merger formation, which makes their electoral performance less subject to large shifts in voters’ preferences.

The main result of the paper shows that when electoral volatility — i.e., the likelihood of large shifts in voters’ preferences — is high enough, in equilibrium parties form strong alliances such as mergers. Intuitively, if voters’ preferences shift too much in one direction, the advantaged party can govern alone; hence for high realizations of the shock the centrist party risks being left out of power. Conversely, as voters’ preferences become more stable, the centrist party values more flexibility and prefers to wait to form a more advantageous

coalition in the future. Electoral instability is often considered a characteristic of the early years of democratic regimes (Kitschelt et al., 1999). This result provides an explanation for the empirical observation that the frequency of mergers decreases as democratic regimes mature (Ibenskas and Sikk, 2017).

How does this central trade-off vary with different electoral, legislative, and executive institutions? The model formalizes how the incentives to form pre-electoral alliances depend on inter-party power sharing (Lijphart, 1984). The degree of power sharing depends on both the rules mapping votes into seats (e.g., electoral rule proportionality) and the rules governing legislative decisions (e.g., the presence of super-majority requirements). Results show that some degree of power concentration is a necessary condition for both PECs and mergers to take place. For example, disproportional electoral systems can induce parties to join forces by forming pre-electoral alliances to maximize their electoral chances (Olsen, 2007; Rakner, Svåsand and Khembo, 2007; Bélanger and Godbout, 2010). Conversely, pre-electoral alliances are not sustainable in consensual democracies that protect minority parties, which feature parties running alone in equilibrium.

While PECs allow parties to campaign autonomously, mergers demand that parties give up their ideological identities by forming new political entities that persist in the future. If voters are uncertain about the exact location of parties' platforms, different configurations of alliances among the same parties might be evaluated differently from the electorate. An extension of the model incorporates voters' uncertainty by introducing noise in the location of parties' platforms. To capture the fact that "mergers reduce, or even destroy, the information value of party labels for voters" (Ibenskas, 2016, 343), I assume that mergers are associated with higher noise than PECs, and the noise is increasing in the distance between the constituent parties' bliss points. The main results are robust to this setting when the noise associated with mergers is not too high. In contrast, mergers are not sustainable in equilibrium for high values of ideological uncertainty.

The paper provides novel insights and implications for the process of party system stabilization. The literature has often linked electoral volatility to unstable party systems. Indeed, several studies even use measures of electoral volatility as an indicator of party system insta-

bility.<sup>5</sup> However, by implicitly assuming that a volatile electorate is responsible for system instability, this approach overlooks the fundamental choices of elites in the determination of party system development (Tavits, 2008). This model suggests to take into account parties' strategic organizational choices to avoid omitted variable bias when evaluating the relation between electoral volatility and party system stability.

The remainder of the paper is organized as follows. Section 2 and 3 present the baseline model and main results. Section 4 and 5 extend the baseline model to consider alternative power sharing institutions and voters' uncertainty over parties' platforms. Section 6 discusses the results and concludes.

## 2. The Model

Consider a two-period game of electoral competition between three policy-motivated parties:  $i = L, C, R$ . Each period features a proposal stage, which determines parties' alliances, and an election. Each party is associated with a preferred policy platform  $z_i \in \mathbb{R}$ , where  $z_l < z_c < z_r$ . There exists a continuum of voters, indexed by  $v$ , who vote for one of the parties. Voters' ideal points are uniformly distributed over a subset of the policy space,  $\mathcal{Z} \equiv [-a, a]$ , where  $\mathcal{Z} \subset \mathbb{R}$ .<sup>6</sup> The ideal policy of voter  $v$  is denoted by  $z_v \in \mathcal{Z}$ .

The sequence of the proposal stage is as follows. First, the centrist party  $C$  proposes to either  $L$  or  $R$  to form a merger, or doesn't propose any merger. If  $C$ 's proposal to  $L$  ( $R$ ) is accepted, the merged party runs against  $R$  ( $L$ ). If  $C$ 's proposal is rejected, or if no merger is proposed,  $C$  proposes a PEC to either party, or doesn't propose any PEC. If  $C$ 's proposal to  $L$  ( $R$ ) is accepted, the PEC formed by  $L, C$  ( $C, R$ ) runs against  $R$  ( $L$ ). If  $C$ 's proposal to  $L$  ( $R$ ) is rejected or if no PEC is proposed, parties compete with their separate identities. After the proposal stage is completed, an election takes place, resulting in the adoption of the policy preferred by the winner.

Notice that the proposal stage rules out the possibility of an alliance between  $L$  and  $R$ . Besides being empirically rare, it is not clear which platform would emerge from an alliance

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<sup>5</sup>Cf. Tavits (2008) for an overview on Western European and OECD countries.

<sup>6</sup>This assumption is without loss of generality and is merely convenient for computing parties' vote shares.

between two non-moderate parties at the opposite extremes of the ideological spectrum, nor how to compute the resulting vote share. The sequence of the proposal is empirically motivated by the flexible nature of PECs vis-à-vis mergers:  $C$  can propose a PEC to either  $L$ ,  $R$  after a merger proposal has been rejected, while it cannot propose a merger to  $L$  ( $R$ ) after a merger proposal to  $R$  ( $L$ ).<sup>7</sup> However, having  $C$  proposing a PEC to only one party or to both does not qualitatively change the results.

In what follows I formally define the policies that result from an alliance between party  $L$  and  $C$ . The policies resulting from an alliance between  $C$  and  $R$  are defined analogously. Denote by  $V_{i,t}$  party  $i$ 's vote share at time  $t$ , where  $t = 1, 2$ . Suppose that  $L$  and  $C$  merge or form a PEC in  $t$ . Then, the policy platform of the resulting party or PEC in  $t$  is a convex combination of the constituent parties' bliss points:

$$z_{l,c,t}^m = z_{l,c,t}^{\text{pec}} = \lambda_{l,t} z_l + (1 - \lambda_{l,t}) z_c. \quad (1)$$

The weight  $\lambda_{l,t} \in (0, 1)$  measures the relative electoral strength of the extreme party ( $L$ ) in  $t$ , which depends on the parties' vote shares as follows:

$$\lambda_{l,t} = \frac{1}{2} + \phi(V_{l,t} - V_{c,t}), \quad (2)$$

where the parameter  $\phi \in \mathbb{R}_+$  is small enough to ensure that  $\lambda_{l,t} \in (0, 1)$ . Equation 1 implies that the policies resulting from PECs and mergers are equivalent *in the same period*.<sup>8</sup>

At the beginning of the second period ( $t = 2$ ), an exogenous shock  $\xi$  favoring party  $R$  affects all voters equally, where  $\xi$  is uniformly distributed in  $[-\frac{1}{\psi}, \frac{1}{\psi}]$ . A positive (negative) realization of the shock shifts voters' ideal points to the right (left). The support of the shock represents electoral volatility: as  $\psi$  decreases, the support of the shock becomes larger, and electoral volatility increases. Conversely, as  $\psi$  increases, the support of the shock shrinks and the electoral outcome becomes more predictable.

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<sup>7</sup>An alternative (less credible) bargaining protocol would allow  $C$  to make sequential merger proposals. However, this would not qualitatively affect the main results.

<sup>8</sup>The extension in Section 5 differentiates between the two types of alliances in the same period by introducing noise in the location of parties' platforms.



After the shock is realized, if no merger formed in  $t = 1$  the proposal and election stages of the second period take place. To simplify the description of the equilibrium, I assume that mergers persist in  $t = 2$  after being formed in  $t = 1$ . That is, constituent parties cannot split in the period that follows the merger formation. This assumption is motivated by the bureaucratic costs and the change in the electorate's preferences that mergers might cause. Typically, several legal requirements are needed for the registration of a new party, which could impede the formation of a splinter party following a recent merger (Hug, 2001). Voters' preferences might also change because of the merger: previous supporters of the constituent parties might transfer their loyalties to the merged party. Furthermore, voters might consider the members of the splinter party as noncredible because of frequent changes in their party affiliation (Mershon and Shvetsova, 2013).

Because of electoral volatility, the policy resulting from a merger (or PEC) formed in  $t = 2$  is different from the policy resulting from a merger formed in  $t = 1$  and persisting in  $t = 2$ . This is because volatility changes parties' relative vote shares and in turn the weight each party has in the common platform. Crucially, while mergers "solidify" the relative power parties have in  $t = 1$  — which is given by each party's vote share  $V_{i,1}$  — PECs are re-negotiated in  $t = 2$ , allowing parties to be flexible to changes in the electoral environment which can alter their relative power.

Voters and parties have standard quadratic preferences over policies. Voter  $v$ 's realized payoff from the implemented policy  $\hat{x}_t$  is defined as  $u_v(\hat{x}_t) = -(z_v - \hat{x}_t)^2$ . Similarly, party  $i$ 's payoff from  $\hat{x}_t$  is  $u_i(\hat{x}_t) = -(z_i - \hat{x}_t)^2$ .

The implemented policy  $\hat{x}_t$  is the preferred policy platform of the winner of the election, i.e., the party, PEC or merger with the majority of votes in  $t$ . If no party/merger/PEC obtains a majority, the implemented policy is determined post-electorally by the party chosen to be the *formateur* — i.e., the party that is awarded the opportunity to form a government. The baseline model assumes that the formateur is the one with the plurality of votes, and that this dominant party (or coalition) can implement its preferred policy after the election. Section 4 analyzes the case where the implemented policy is a compromise among the policy positions

of all the parties composing the parliament, without regard to whether these parties are in government or opposition.

The timing of the game is as follows:

1. The first period proposal and election stages take place, and the policy outcome is implemented.
2. Nature determines the realization of the shock to voters' preferences.
3. If a merger occurred in the first period, the second period election takes place. If no merger occurred in the first period, the second period proposal and election stages take place, and the policy outcome is implemented.

I focus on subgame perfect equilibria in pure strategies. For party  $C$  a pure strategy is a proposal decision in  $t = 1$  and, conditional on no mergers forming in  $t = 1$ , a proposal decision in  $t = 2$ . For party  $L$  ( $R$ ) a pure strategy is an acceptance decision in  $t = 1$  and, conditional on no mergers forming in  $t = 1$ , an acceptance decision in  $t = 2$ . Since no voter is ever pivotal, I adopt the standard assumption that voters vote sincerely. Furthermore, I assume that voters maximize their current period payoff in each election. Parties, on the other hand, maximize their expected overall payoff, and each party evaluates the future according to a common discount factor  $\delta \in (0, 1)$ .

The following analysis assumes without loss of generality that  $C$  is weakly closer to  $L$  than to  $R$ :  $|z_c - z_l| \leq |z_c - z_r|$ . To avoid trivialities I also assume that in the first period i) no party has an outright majority and ii) parties' ideal points are such that  $C$  does not have a plurality of votes and would obtain a majority by forming either alliance (with  $L$  or  $R$ ).<sup>9</sup>

### 3. Analysis

I start by computing the voters' decision in the second period, which determines the vote share of each party. Given these vote shares, I analyze parties' decision to form a merger or PEC or to run alone. Given the second period outcomes, I compute the expected payoff of

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<sup>9</sup>Notice that these assumptions imply that in the first period  $z_c \in \mathcal{Z}$ , while  $z_l$  and/or  $z_r$  can lie outside of  $\mathcal{Z}$ .

each party from merging, forming a PEC or running alone in the first period as a function of electoral volatility, and characterize the equilibrium of the game.

### 3.1. Pre-Electoral Coalitions

Let us analyze first what happens in the second period *when no merger formed in the first period*. To compute party  $i$ 's vote share from running alone ( $V_{i,2}$ ) it suffices to identify the location of the voter who is indifferent between each pair of parties. Let  $v_{lc,2}$  denote the ideal point of the voter who is indifferent between  $L$  and  $C$  in  $t = 2$ , where  $v_{lc,2}$  is located at  $(z_l + z_c)/2$ . The voter who is indifferent between  $C$  and  $R$ , denoted by  $v_{cr,2}$ , is defined analogously. Then, the vote share of  $L$  is the CDF of the distribution of voters' ideal points evaluated at  $v_{lc,2}$ . Since voters' bliss points are uniformly distributed on  $\mathcal{Z}$ ,  $L$ 's vote share is simply:

$$V_{l,2} = \frac{2a + z_l + z_c - 2\xi}{4a}, \quad (3)$$

which depends on the realization of the shock to voters' preferences. A positive (negative) realization of the shock shifts voters' ideal policies to the right (left) thereby increasing the vote share of party  $R$  ( $L$ ) by  $|\xi|$ . Similarly,  $V_{c,2} = (z_r - z_l)/4a = V_{c,1}$  and

$$V_{r,2} = 1 - V_{l,2} - V_{c,2} = \frac{2a - z_c - z_r + 2\xi}{4a}. \quad (4)$$

The vote share of a PEC formed in the second period is derived analogously. Let  $V_{lc,2}^{\text{pec}}$  be the vote share of a PEC between  $L$  and  $C$  in  $t = 2$ . Similarly to  $V_{l,2}$  (3), the PEC's vote share is computed by finding the location of the voter who is indifferent between  $\lambda_{l,2} z_l + (1 - \lambda_{l,2}) z_c$  and  $z_r$ , which produces

$$V_{lc,2}^{\text{pec}} = \frac{8a^2 + 2a(z_c + z_l + 2z_r - \phi z_c + \phi z_l - 4\xi) - \phi(z_c - z_l)(z_c - z_r + 2(z_l - \xi))}{16a^2}. \quad (5)$$

Similarly, the vote share of a PEC between  $C$  and  $R$  is

$$V_{cr,2}^{\text{pec}} = \frac{8a^2 - 2a(z_c + z_r + 2z_l - \phi z_c + \phi z_r - 4\xi) - \phi(z_c - z_r)(z_c - z_l + 2(z_r - \xi))}{16a^2}. \quad (6)$$

Finally, recall that  $z_{lc,2}^m = z_{lc,2}^{\text{pec}}$  (1), which implies that the vote share of a merger formed in  $t = 2$  is analogous to that of a PEC: i.e.,  $V_{lc,2}^m = V_{lc,2}^{\text{pec}}$  and  $V_{cr,2}^m = V_{cr,2}^{\text{pec}}$ .

Given these vote shares, what determines parties' choice in the second period? In the proposal stage, parties compare the realized payoff from merging, forming a PEC, and running alone. Because  $z_{lc,2}^m = z_{lc,2}^{\text{pec}}$  and  $z_{cr,2}^m = z_{cr,2}^{\text{pec}}$ , parties are indifferent between merging and forming a PEC in  $t = 2$ . I assume that, when indifferent, party  $i$  chooses a PEC. It follows that party  $i$  compares the realized payoffs from the two possible PECs to that of running alone. These payoffs depend on the location of parties' ideal points, and on the realization of the shock to voters' preferences.

The shock has a twofold impact on parties' decision: first, it has a *direct* effect on parties' vote share, by swinging voters' preferences in favor of either  $L$  or  $R$ . I denote this the *electoral effect*. Second, by changing parties' relative vote share, the shock *indirectly* affects parties' influence on the final policy of a PEC. I denote this the *policy effect*.

In what follows I define threshold values of the shock realization that determine which of these two effects prevails in parties' decision to form a PEC in  $t = 2$ . These values also provide useful cutoffs to describe parties' equilibrium behavior in the second period.

**Definition 1.** Let  $\underline{\xi}(z_l, z_c, z_r)$  be the value of  $\xi$  such that  $L$ 's vote share  $V_{l,2} > 1/2$  for  $\xi < \underline{\xi}(z_l, z_c, z_r)$ . It follows from the expression of  $V_{l,2}$  (3) that  $\underline{\xi} = \frac{z_l + z_c}{2}$ .

Similarly, let  $\bar{\xi}(z_l, z_c, z_r)$  be the value of the shock realization such that  $R$ 's vote share  $V_{r,2} > 1/2$  for  $\xi > \bar{\xi}(z_l, z_c, z_r)$ . It follows from the expression of  $V_{r,2}$  (4) that  $\bar{\xi} = \frac{z_c + z_r}{2}$ .

Let us first consider parties' decision when  $\xi > \bar{\xi}$ . When a party has the majority of votes, the electoral effect trumps every other consideration: by running alone,  $R$  can implement its preferred policy. Similarly, when  $\xi < \underline{\xi}$  party  $L$  runs alone and wins, hence the implemented policy is  $\hat{x}_2 = z_l$ . Hence, for  $\xi < \underline{\xi}$  ( $\xi > \bar{\xi}$ )  $L$  ( $R$ ) rejects a PEC proposal from  $C$  and in equilibrium parties run alone in the second period.

When  $\underline{\xi} < \xi < \bar{\xi}$ , no party obtains an absolute majority if all parties run alone, yet a party that runs alone against a PEC could obtain a majority of votes. In particular, when parties form PECs, it could be that (i)  $V_{lc,2}^{\text{pec}} > 1/2$ , (ii)  $V_{cr,2}^{\text{pec}} > 1/2$ , or both. The following definition derives values of the shock realization that define each of these occurrences.

**Definition 2.** Let  $\underline{\xi}^{pec}(z_l, z_c, z_r)$  be the value of  $\xi$  such that  $V_{cr,2}^{pec} > 1/2$  for  $\xi > \underline{\xi}^{pec}(z_l, z_c, z_r)$ . It follows from the expression of  $V_{cr,2}^{pec}$  (6) that

$$\underline{\xi}^{pec} = \frac{2a(z_c + (z_r - z_c)\phi + z_r + 2z_l) + \phi(z_c - z_r)(z_c + 2z_r - z_l)}{8a + 2\phi(z_c - z_r)}. \quad (7)$$

Similarly, let  $\bar{\xi}^{pec}(z_l, z_c, z_r)$  be the value of  $\xi$  such that  $L$ 's vote share  $V_{lc,2}^{pec} > 1/2$  for  $\xi < \bar{\xi}^{pec}(z_l, z_c, z_r)$ . It follows from the expression of  $V_{lc,2}^{pec}$  (5) that

$$\bar{\xi}^{pec} = \frac{2a(z_c + (z_l - z_c)\phi + z_l + 2z_r) - \phi(z_c - z_l)(z_c + 2z_l - z_r)}{8a + 2\phi(z_l - z_c)}. \quad (8)$$

Let us analyze  $C$ 's decision when  $\underline{\xi}^{pec} < \xi < \bar{\xi}^{pec}$ . Definition 2 implies that for these values of the shock realization both PECs would reach an absolute majority. Then,  $C$ 's proposal determines which PEC is formed in equilibrium. Under the assumptions, both  $L$  and  $R$  accept  $C$ 's proposal — as running alone would result in a certain loss — and in  $t = 2$  a PEC is formed. Then,  $C$ 's decision determines whether the PEC is between  $L$  and  $C$  or between  $C$  and  $R$ .<sup>10</sup>  $C$  compares the payoff from forming a PEC with  $L$ , i.e.,

$$u_c(z_{lc,2}^{pec}) = -\frac{(z_c - z_l)^2 [\phi(z_c - z_r + 2(z_l - \xi)) + 2a(\phi + 1)]^2}{16a^2}, \quad (9)$$

with the payoff from forming a PEC with  $R$

$$u_c(z_{cr,2}^{pec}) = -\frac{(z_c - z_r)^2 [\phi(z_c - z_l + 2(z_r - \xi)) - 2a(\phi + 1)]^2}{16a^2}. \quad (10)$$

The following results show how  $C$ 's decision changes with different values of the shock realization and with the location of parties' platforms. In particular, Lemma 1 shows that, as voters' preferences shift in favor of  $R$  ( $L$ ), the centrist party prefers a coalition with  $L$  ( $R$ ). Lemma 2 then shows that  $C$  prefers an alliance with the ideologically closest party when

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<sup>10</sup>Running alone is strictly dominated for  $C$ , because it would result in the adoption of the policy preferred by the party with the plurality of votes.

voters' preferences are stable (i.e.,  $\xi = 0$ ). Finally, Proposition 1 characterizes the (second period) equilibrium alliance configuration based on the value of the shock realization.

**Lemma 1.** Policy Effect. Let  $\Delta_c^{pec}(\xi) = u_c(z_{lc,2}^{pec}) - u_c(z_{cr,2}^{pec})$ .  $\Delta_c^{pec}(\xi)$  is strictly increasing in  $\xi$ .

**Proof.** All proofs can be found in the Appendix. □

When  $\underline{\xi}^{pec} < \xi < \bar{\xi}^{pec}$  both PECs obtain a majority if formed. When this is the case, Lemma 1 shows that the policy effect determines  $C$ 's proposal decision. To see why, suppose that the shock realization is such that  $C$  is indifferent between the two coalitions. Now, let the value of the shock realization increase. This increase leads to a higher (lower) vote share of party  $R$  ( $L$ ), which means that  $R$  ( $L$ )'s preferred policy weighs more (less) in a PEC between  $C$  and  $R$  ( $L$ ). Then, ceteris paribus,  $C$  would prefer to form a PEC with  $L$ . Conversely, a lower value of the shock makes a coalition with  $R$  more appealing.

Whether  $C$  forms a PEC with  $L$  or  $R$  ultimately depends on the location of the platforms  $z_i$ . Let us first evaluate what PEC party  $C$  prefers when  $\xi = 0$ , i.e., when voters' preferences are stable. In this case,  $C$  is indifferent between  $L$  and  $R$  (i.e.,  $\Delta_c^{pec}(0) = 0$ ) when  $z_l$  and  $z_r$  are equidistant from  $z_c$ , and prefers the closer ally otherwise, as the next result shows.

**Lemma 2.**  $\Delta_c^{pec}(0)$  is strictly increasing in  $z_r$ .

Since  $C$  is closer to  $L$  than to  $R$  by assumption, a corollary of Lemma 2 is that when  $\xi = 0$  party  $C$  prefers a coalition with  $L$ . Furthermore, Lemma 1 implies that when the shock favors  $R$  (i.e., when  $\xi > 0$ ),  $C$  continues to prefer an alliance with  $L$ .

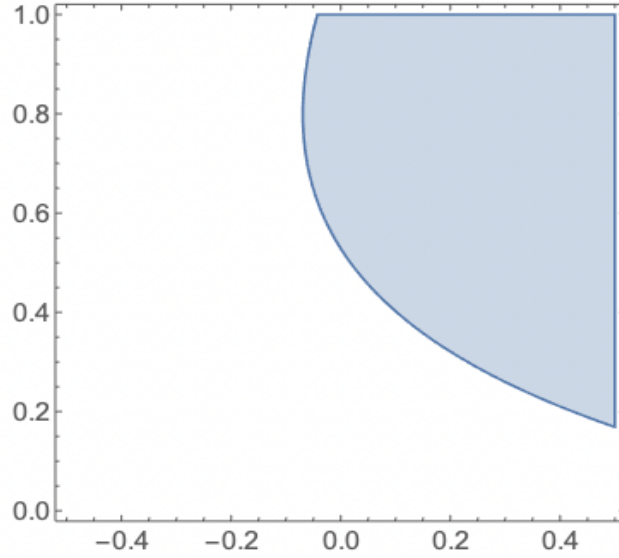
The next definition derives the value of the shock realization,  $\hat{\xi}$ , such that party  $C$  is indifferent between proposing a PEC to  $L$  or  $R$  (i.e.,  $\Delta_c^{pec}(\hat{\xi}) = 0$ ) for any  $z_i$ .

**Definition 3.** Let  $\hat{\xi}(z_l, z_c, z_r)$  be the value of the shock realization such that  $\Delta_c^{pec}(\hat{\xi}) = 0$ . It follows from the expression of  $\Delta_c^{pec}$  (9-10) that

$$\hat{\xi} = \frac{a(\phi + 1)(2z_c - z_l - z_r)}{\phi(z_l - z_r)} - z_c + z_l + z_r. \quad (11)$$

It follows from Lemma 1 that  $C$  prefers to form a PEC with  $L$  ( $R$ ) when  $\xi > \hat{\xi}$  ( $\xi < \hat{\xi}$ ). Whenever both PECs obtain the majority of votes ( $\underline{\xi}^{\text{pec}} < \xi < \bar{\xi}^{\text{pec}}$ ), the threshold  $\hat{\xi}$  determines which of the two PECs form.

Figure 1 summarizes the implications of Lemma 1 and Lemma 2, plotting the region such that  $\Delta_c^{\text{pec}}(\xi) > 0$  as a function of  $\xi$  (x axis) and  $z_r$  (y axis). Party  $L$  and  $C$ 's preferred platforms are set respectively to  $z_l = -0.6$  and  $z_c = 0$ .



**Figure 1 – PEC decision.**  $\Delta_c^{\text{pec}}(\xi)$  as a function of the value of  $\xi$  (x axis) and  $z_r$  (y axis). The blue region corresponds to the values of  $\xi$ ,  $z_r$  such that  $C$  prefers a coalition with  $L$  than with  $R$  ( $\Delta_c^{\text{pec}} > 0$ ). The other parameters are set to  $z_l = -0.6$ ,  $z_c = 0$ ,  $a = 1$  and  $\phi = 1.5$ .

When the electoral shock favors  $R$  ( $\xi > 0$ , right region), party  $C$  prefers to form a PEC with  $L$ , unless  $R$  is ideologically close enough. When the shock realization favors  $L$  ( $\xi < 0$ , left region), party  $C$  prefers to form a PEC with  $R$ . This happens because the policy cost effect from a PEC with  $L$  induces the centrist party to form a coalition with  $R$  (Lemma 1). This policy effect prevails whenever  $C$  could achieve a majority by forming a PEC with both parties (i.e., when  $\underline{\xi}^{\text{pec}} < \xi < \bar{\xi}^{\text{pec}}$ ). Fix  $z_r = 0.7$ . For these parameter values, we have that  $\underline{\xi}^{\text{pec}} = -0.17$ ,  $\bar{\xi}^{\text{pec}} = 0.24$ , and that  $\hat{\xi} = -0.03$ . Hence, in equilibrium a PEC between  $C$  and  $R$  ( $C$  and  $L$ ) forms for  $\underline{\xi}^{\text{pec}} < \xi < \hat{\xi}$  ( $\hat{\xi} < \xi < \bar{\xi}^{\text{pec}}$ ).

Finally, it could be that only one PEC has the absolute majority of votes in the second period. Suppose that  $V_{lc,2}^{pec} > 1/2$  and  $V_{cr,2}^{pec} < 1/2$ .<sup>11</sup> If  $C$  were to propose a PEC to  $L$ ,  $L$  would reject because it could set its preferred platform by forming a minority government after elections.<sup>12</sup> Similarly, because  $L$  has a relative majority, a PEC between  $C$  and  $R$  would not change the post-electoral policy set by  $L$ . Hence, when only a PEC between  $L$  and  $C$  reaches the absolute majority of votes, in equilibrium parties run alone and  $L$  forms a minority government (the case such that  $V_{cr,2}^{pec} > 1/2$  is analogous).

The following proposition summarizes the last observation and the previous results without proof.

**Proposition 1.** *PEC Decision and Second-Period Policy Outcome. Suppose that no merger formed in  $t = 1$ . Then, in  $t = 2$  parties form PECs for intermediate realizations of the shock  $\xi$ , and compete alone for extreme ones. In particular, for  $\underline{\xi}^{pec} < \xi < \hat{\xi}$  ( $\hat{\xi} < \xi < \bar{\xi}^{pec}$ ), a PEC between  $C, R$  ( $C, L$ ) forms, and  $\hat{x}_2 = z_{cr,2}^{pec}$  ( $z_{lc,2}^{pec}$ ). Conversely, when  $\xi < \underline{\xi}^{pec}$  ( $\xi > \bar{\xi}^{pec}$ ), parties run alone and  $\hat{x}_2 = z_l$  ( $z_r$ ).*

The second-period analysis summarized in Proposition 1 suggests when we should expect parties to run alone or to form alliances. One interesting insight that emerges from the analysis is that parties can join PECs to prevent other parties from obtaining an absolute majority and control of the policy-making process. In line with this logic, [Hortala-Vallve, Meriläinen and Tukiainen \(2021\)](#) provide evidence from Finnish municipalities that parties join PECs to avoid concentration of power in the hands of the largest party when this is close to obtaining more than half of the seats. Similarly, [Frey, López-Moctezuma and Montero \(2021\)](#) document that in Mexican mayoral elections parties form alliances to remove advantaged incumbent parties from office. Furthermore, taking the electoral effect as given, Lemma 2 suggests that we should expect PECs to be more likely among ideologically close parties, a result which is widely supported by the empirical literature on pre-electoral coalitions ([Golder, 2006](#); [Ibenskas, 2016](#); [Hortala-Vallve, Meriläinen and Tukiainen, 2021](#)).

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<sup>11</sup>It follows from Definition 2 that this is the case for  $\underline{\xi} < \xi < \underline{\xi}^{pec}$ .

<sup>12</sup>Notice that this would not be true if the realized policy was determined after parliamentary negotiations among all parties. Section 4 analyzes this possibility.



Let  $U_{i,2}(\neg m_1)$  denote the expected second-period payoff of party  $i$ , when no merger formed in the first period. Proposition 1 allows us to express  $U_{i,2}(\neg m_1)$  as a function of electoral volatility. By the uniform assumption of the shock, the probability of  $\xi$  falling below some threshold  $x$  is  $\Pr\{\xi < x\} = \frac{1}{2} + \frac{\psi}{2}(x)$ , hence the expected payoff from the second period is simply:

$$U_{i,2}(\neg m_1) = \left[ \frac{1}{2} + \frac{\psi}{2}(\underline{\xi}^{\text{pec}}) \right] u_i(z_l) + \left[ \frac{\psi}{2}(\hat{\xi}) - \frac{\psi}{2}(\underline{\xi}^{\text{pec}}) \right] V_{i,2}(z_{cr,2}^{\text{pec}}) \\ + \left[ \frac{\psi}{2}(\bar{\xi}^{\text{pec}}) - \frac{\psi}{2}(\hat{\xi}) \right] V_{i,2}(z_{lc,2}^{\text{pec}}) + \left[ \frac{1}{2} - \frac{\psi}{2}(\bar{\xi}^{\text{pec}}) \right] u_i(z_r), \quad (12)$$

where  $V_{i,2}(z_{lc,2}^{\text{pec}})$  is the expected payoff of party  $i$  from the LC coalition platform, which depends on the realization of the shock:

$$V_{i,2}(z_{lc,2}^{\text{pec}}) = \int_{\hat{\xi}}^{\bar{\xi}^{\text{pec}}} u_i(z_{lc,2}^{\text{pec}}) \frac{1}{\bar{\xi}^{\text{pec}} - \hat{\xi}} d\xi, \quad (13)$$

and analogously for  $V_{i,2}(z_{cr,2}^{\text{pec}})$ . These expressions will determine the equilibrium in the first period, when parties compare  $U_{i,2}(\neg m_1)$  to the expected second-period payoff conditional on a merger in  $t = 1$ , which is derived next.

### 3.2. Mergers

The second-period analysis following a merger in  $t = 1$  is more straightforward. Suppose that a merger between  $L$  and  $C$  formed. Let  $\tilde{\xi}_l$  be the value of the shock realization such that a merger between  $L$  and  $C$  obtains half of the vote share, where  $\tilde{\xi}_l = (z_{lc}^m + z_r)/2$ . Then, for  $\xi < \tilde{\xi}_l$ , the policy outcome is  $\hat{x}_2 = z_{lc}^m$ , otherwise it is  $\hat{x}_2 = z_r$ . Similarly, suppose that a merger between  $C$  and  $R$  formed in  $t = 1$ . Let  $\tilde{\xi}_r$  be the value of the shock realization such that a merger between  $C$  and  $R$  obtains half of the vote share, where  $\tilde{\xi}_r = (z_l + z_{cr}^m)/2$ . For  $\xi > \tilde{\xi}_r$ , the policy outcome is  $\hat{x}_2 = z_{cr}^m$ , otherwise it is  $\hat{x}_2 = z_l$ .

Denote by  $U_{i,2}(m_{lc,1})$  the expected second-period payoff of party  $i$ , when a merger between  $L$  and  $C$  formed in the first period. This is simply

$$U_{i,2}(m_{lc,1}) = \left[ \frac{1}{2} + \frac{\psi}{4}(z_{lc,1}^m + z_r) \right] u_i(z_{lc,1}^m) + \left[ \frac{1}{2} - \frac{\psi}{4}(z_{lc,1}^m + z_r) \right] u_i(z_r). \quad (14)$$

Similarly, the expected payoff of party  $i$  from a merger between  $C$  and  $R$  can be written as

$$U_{i,2}(m_{cr,1}) = \left[ \frac{1}{2} - \frac{\psi}{4}(z_l + z_{cr,1}^m) \right] u_i(z_{cr,1}^m) + \left[ \frac{1}{2} + \frac{\psi}{4}(z_l + z_{cr,1}^m) \right] u_i(z_l). \quad (15)$$

Given these expressions, we can easily compare party  $C$ 's expected payoff from merging with  $L$  and  $R$ . The expected payoff of party  $i$  from a merger between  $L$  and  $C$  is the sum of two components: the realized payoff from the merged party policy platform in  $t = 1$  and the expected payoff from the winning policy in  $t = 2$  following a merger (14), i.e.,

$$U_{i,lc}^m = u_i(z_{lc,1}^m) + \delta U_{i,2}(m_{lc}), \quad (16)$$

where the realized policy in the first period coincides with the merged party's platform, since the merger has the majority of votes in  $t = 1$ . The expression for  $U_{i,cr}^m$  is analogous.

When does  $C$  prefer to merge with the closest party  $L$ ? The payoff that  $C$  obtains in the first period from merging with  $L$  is clearly higher than the one following a merger with  $R$ , because the implemented policy resulting from the former is closer to  $C$ 's ideal point. Yet, depending on the probability of winning the election in the second period,  $C$  might prefer to merge with  $R$ . The next result shows that as volatility increases  $C$  prefers to merge with the ideologically more distant party ( $R$ ), which benefits more from a volatile electorate than  $L$ .

**Lemma 3.** *Let  $\Delta_c^m(\psi) = U_{i,lc}^m - U_{i,cr}^m$ .  $\Delta_c^m(\psi)$  is strictly increasing in  $\psi$ .*

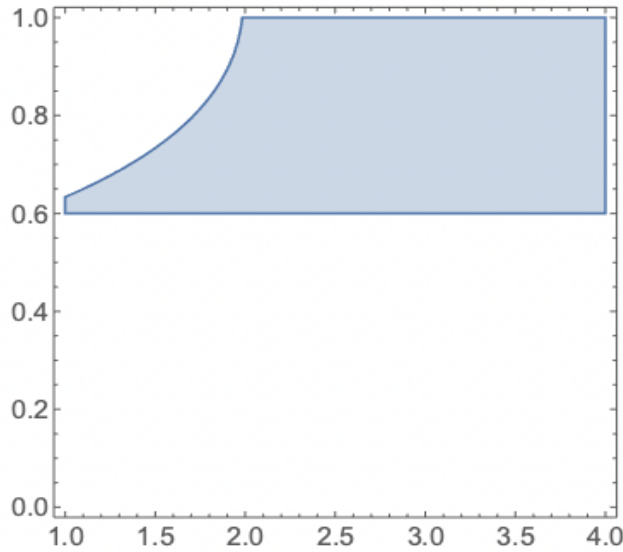
Similarly to the second period analysis over PECs, whether  $C$  prefers to merge with  $L$  or  $R$  depends on the location of the platforms. Intuitively, as  $R$  moves away from  $C$ 's preferred platform,  $C$  is more likely to form a merge with  $L$ , as shown in the next result.

**Lemma 4.**  *$\Delta_c^m(\psi)$  is strictly increasing in  $z_r$ .*

It follows from Lemma 3 and Lemma 4 that, depending on the value of electoral volatility,  $C$  might prefer to merge with either  $L$  or  $R$ . Let  $\tilde{\psi}(z_l, z_c, z_r)$  be the value of the shock realization such that  $\Delta_c^m(\tilde{\psi}) = 0$ , that is, such that party  $C$  is indifferent between proposing a merger to  $L$

or  $R$  in the first period for any  $z_i$ .<sup>13</sup> Then,  $C$  prefers to form a merger with  $R$  ( $L$ ) when  $\psi < \tilde{\psi}$  ( $\psi > \tilde{\psi}$ ).

Figure 2 summarizes these observations, plotting the region such that  $\Delta_c^m(\psi) > 0$  as a function of  $\psi$  (x axis) and  $z_r$  (y axis). Party  $L$  and  $C$ 's preferred platforms are set to  $z_l = -0.6$  and  $z_c = 0$  respectively. Intuitively, when  $z_r$  is closer to  $C$  than  $L$  ( $z_r < 0.6$ ),  $C$  prefers to merge with  $R$ . As  $R$  becomes more extreme than  $L$ , which merger is preferred from  $C$  depends on electoral uncertainty: as the support of the shock grows,  $C$  can prefer a merger with  $R$  (upper left region), even if the latter is further away from  $C$ . In other words, by affecting the future expected vote share electoral volatility can mute policy considerations when comparing mergers with different parties in the first period.



**Figure 2 – Merger decision.**  $\Delta_c^m(\psi)$  as a function of the value of  $\psi$  (x axis) and  $z_r$  (y axis). The blue region corresponds to the values of  $\psi$ ,  $z_r$  such that  $C$  prefers to merge with  $L$  rather than with  $R$  ( $\Delta_c^m > 0$ ). The other parameters are set to  $z_l = -0.6$ ,  $z_c = 0$ ,  $a = 1$  and  $\phi = 1.5$ .

### 3.3. When are Mergers Sustainable? The Role of Electoral Volatility

It is now possible to describe the equilibrium of the game. The next result shows that there exists a trade-off between merging and forming a PEC depending on electoral volatility.

<sup>13</sup>The expression for  $\tilde{\psi}(z_l, z_c, z_r)$  is presented in the Appendix.

The previous section has derived the expected payoff of party  $i$  from a merger between  $L$  and  $C$  (16). This is compared to the expected payoff of party  $i$  from a coalition between  $L$  and  $C$ , i.e.:

$$U_{i,lc}^{\text{pec}} = u(z_{lc,1}^{\text{pec}}) + \delta U_{i,2}(-m_1), \quad (17)$$

where the second component of the RHS is party  $i$ 's expected payoff in  $t = 2$  following a PEC between  $L$  and  $C$  (12). The expressions for  $U_{i,cr}^{\text{pec}}$  is analogous, where the first-period realized payoff is  $u_i(z_{cr}^{\text{pec}})$ , and the second period expected payoff is  $U_{i,2}(-m_1)$ .

What conditions can sustain an equilibrium in which parties merge? For  $C$  to prefer a merger with  $L$ , it must be that (i)  $U_{c,lc}^m > U_{c,cr}^m$ , (ii)  $U_{c,lc}^m > U_{c,lc}^{\text{pec}}$ , (iii)  $U_{c,lc}^m > U_{c,cr}^{\text{pec}}$  and (iv)  $U_{c,lc}^m > U_c^{\text{alone}}$ .<sup>14</sup> Notice that we know from Lemma 3 that for high electoral volatility  $C$  prefers to merge with the more extreme party  $R$ . Furthermore, we can immediately compare the expected payoff from the two PECs, because the second period payoff is the same for both of them (12). This leads to the following strict ranking for party  $C$ :  $U_{c,lc}^{\text{pec}} > U_{c,cr}^{\text{pec}}$ , which simply follows from comparing the first-period payoffs.<sup>15</sup>

The next result describes the equilibrium of the baseline game, showing that different alliance configurations can emerge depending on the electorate's volatility.

**Proposition 2.** Electoral Volatility and Merger Equilibrium. *Let  $\hat{\psi}$  be the value of  $\psi$  such that  $C$  is indifferent between merging and forming a PEC with the closest party ( $L$ ). In the first period, when electoral volatility is sufficiently low ( $\psi > \hat{\psi}$ ),  $C$  forms a PEC with the closest party ( $L$ ). Mergers emerge for high electoral volatility: when  $\tilde{\psi} < \psi < \hat{\psi}$ ,  $C$  merges with the closest party ( $L$ ), and when  $\psi < \tilde{\psi}$ ,  $C$  merges with the more extreme party ( $R$ ).*

Proposition 2 conveys a simple intuition about parties' incentives to join different types of alliances. When the likelihood of large shifts in voters' preferences is high enough, in equilibrium the centrist party prefers to merge rather than to form a PEC. By merging, the centrist party insures itself against large shifts in the electorate's preferences, at the cost of losing the opportunity to form a more advantageous coalition in the future. Furthermore,

<sup>14</sup>Clearly, conditions (i)-(iv) are necessary but not sufficient for a merger between  $C$  and  $L$  to form in equilibrium, as the merger must be incentive compatible for  $L$  as well.

<sup>15</sup>It is straightforward to derive similar rankings for  $L$  and  $R$  respectively:  $U_{l,lc}^{\text{pec}} > U_{l,cr}^{\text{pec}}$  and  $U_{r,cr}^{\text{pec}} > U_{r,lc}^{\text{pec}}$ .

when  $\tilde{\psi} < \psi < \hat{\psi}$ ,  $C$  chooses the ideologically closest party to minimize the policy cost from the merged party platform. This result is consistent with empirical evidence on the ideological location of constituent parties joining mergers (Ibenskas, 2016). However, when volatility is extremely high ( $\psi < \tilde{\psi}$ ), electoral considerations might trump the policy effect, resulting in a merger with more distant allies.

Proposition 2 also shows that mergers are not sustainable anymore when voters' preferences are stable — which can be empirically associated with a highly partisan electorate. In this case, the centrist party values more flexibility, and forms with the closest party a temporary alliance which does not bind its policy platform in the future. By forming a PEC in the first period, the centrist party maintains its original platform, preserving its brand for the future election, when more information about voters' preferences is available.

Proposition 2 suggests that we should expect mergers to be empirically associated with volatile electorates. In principle, an accurate measure of electoral volatility should reflect the extent to which personal votes change between subsequent elections. Thus, individual level data identifying voters' intentions to vote or party identification across time represent an accurate measure of volatility.

In the absence of individual level data, empirical analysts have turned to aggregate measures of volatility (Pedersen, 1979; Bartolini and Mair, 1990; Sikk, 2005; Emanuele, 2015). The original index of volatility, developed by Pedersen (1979), measures the sum of the absolute values of vote percentage changes of parties from one election to another divided by two. This measure presents endogeneity concerns, as mergers alter the configuration of the party system thus generating volatility. One solution, as suggested by Sikk (2005), is to consider the merged parties as one in the election where they ran separately. This approach is preferred because it does not lead to overestimation of volatility scores. It is a conservative approach because it assumes that the constituent parties' voters should also support the merged party, thus underestimating voter mobility.

### 3.4. Illustration: Equilibrium Alliances and Volatility

Proposition 2 shows that a merger between  $C$  and  $L$  is only sustainable in equilibrium when  $U_{c,cr}^m > U_{c,lc}^{pec}$ , as in this case a merger is incentive compatible for  $L$ . Conversely, when  $U_{c,cr}^m <$

$U_{c,lc}^{\text{pec}}$ ,  $L$  could reject a merger proposal and the outcome would be a PEC with  $C$ ,  $L$ 's preferred option. The following example illustrates this point, by deriving the equilibrium for fixed parties' preferred platforms and showing each party's incentives to form alliances given different values of electoral uncertainty.

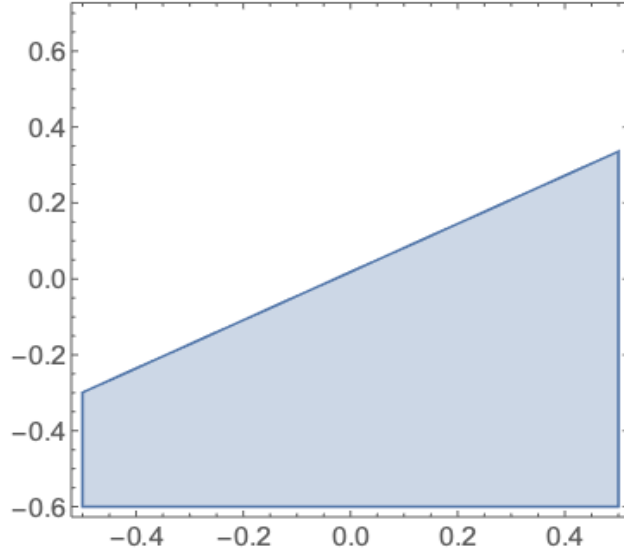
Considers parties' platforms such that the centrist party lies in the middle of the policy space  $\mathcal{Z} = [-1, 1]$ , and the right party is more extreme than the left. Let  $z_l = -0.6$ ,  $z_c = 0$  and  $z_r = 0.7$ , such that no party has a majority in  $t = 1$ :  $V_{l,1} = 0.35$ ,  $V_{c,1} = 0.325$ ,  $V_{r,1} = 0.325$ .

Let's start by computing parties' decision in the second period. For high values of the shock realization ( $\xi > \bar{\xi}$ )  $R$  runs alone and the implemented policy is  $\hat{x}_2 = z_r$ . Similarly, for low values of the shock realization ( $\xi < \underline{\xi}$ )  $L$  runs alone and  $\hat{x}_2 = z_l$ . For intermediate values of the shock realization ( $\underline{\xi}^{\text{pec}} < \xi < \bar{\xi}^{\text{pec}}$ ),  $V_{lc,2}^{\text{pec}} > 1/2$  and  $V_{cr,2}^{\text{pec}} > 1/2$ . In this case, both  $L$  and  $R$  are willing to form an alliance with  $C$ , and in  $t = 2$  a PEC is formed.

To decide which PEC to form,  $C$  compares the payoff from a PEC with  $L$  (9) to that of a PEC with  $R$  (10). Whether one alliance is preferred to the other depends on the value of the shock realization. Recall that  $\Delta_c^{\text{pec}}(\xi) = u_c(z_{lc,2}^{\text{pec}}) - u_c(z_{cr,2}^{\text{pec}})$ . Figure 3 provides a graphical representation of  $C$ 's decision, plotting the region for which  $\Delta_c^{\text{pec}}(\xi) > 0$  as a function of the shock realization (x axis) and  $z_c$  (y axis), for  $z_l = -0.6$  and  $z_r = 0.7$ . As the shock favors  $R$ ,  $C$ 's incentives to form a coalition with  $L$  increase because of the policy effect of an increased weight in the PEC platform.

For these parameter values, the following is the equilibrium second-period outcome as a function of the shock realization: when  $\xi < \underline{\xi} = -0.17$ , parties run alone and  $\hat{x}_2 = z_l$ , when  $\underline{\xi} < \xi < \hat{\xi}$  (where  $\hat{\xi} = -0.03$ ) a PEC among  $C$  and  $R$  forms and  $x_2 = z_{cr,2}^{\text{pec}}$ , when  $\hat{\xi} < \xi < \bar{\xi}$  (where  $\bar{\xi} = 0.24$ ) a PEC among  $C$  and  $L$  forms and  $\hat{x}_2 = z_{lc,2}^{\text{pec}}$ , and when  $\xi > \bar{\xi}$ , parties run alone and  $\hat{x}_2 = z_r$ .

In the first period parties compare the expected values of merging, forming PECs and running alone as a function of electoral volatility. Figure 4 illustrates which types of alliances emerge in equilibrium as a function of electoral volatility. The orange and gray regions plot the range of parameters sustaining an equilibrium where parties merge in the first period,

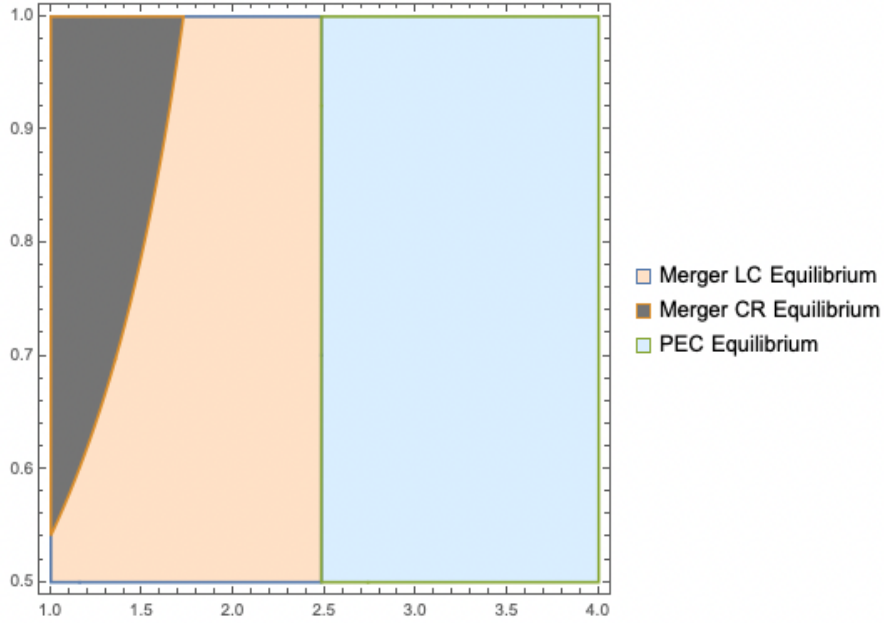


**Figure 3** –  $\Delta_c^{\text{pec}}(\xi)$  as a function of the value of  $\xi$  (x axis) and  $z_c$  (y axis). The blue region corresponds to the values of  $\xi, z_c$  such that  $C$  prefers a coalition with  $L$  than with  $R$  ( $\Delta_c^{\text{pec}} > 0$ ). The other parameters are set to  $z_l = -0.6, z_r = 0.7, a = 1, \psi = 2$  and  $\phi = 1$ .

while the blue region plots the range for which parties form PECs in the first period, as a function of  $\psi$  (x axis) and parties' discount factor (y axis).

Which type of alliance between  $C$  and  $L$  is sustainable in equilibrium, for these parameter values? To answer, we need to verify that a merger (or PEC) is incentive compatible for  $L$  for some of the values of volatility for which  $C$  wants to merge (or form a PEC) with  $L$ . For the parameter values in this example,  $L$  always prefers a PEC to a merger with  $C$ . Hence, when volatility is low ( $\psi$  high enough),  $C$  proposes a PEC to  $L$ , which accepts, and a PEC forms in equilibrium (blue region).

As electoral volatility increases ( $\psi$  decreases), the centrist party's incentives to merge increase. Suppose that  $C$  proposes a merger to  $L$  for  $\tilde{\psi} < \psi < \hat{\psi}$ , i.e., for the values of volatility such that  $C$  prefers a merger with  $L$  to both a merger with  $R$  and to a PEC with  $L$ . If  $L$  accepts, its expected payoff is  $U_{l,lc}^m$ . If  $L$  rejects, the outcome depends on  $C$ 's ranking of alternatives, which varies with  $\psi$ . That is, if  $U_{c,lc}^m > U_{c,lc}^{\text{pec}} > U_{c,cr}^m$ , knowing  $C$ 's ranking  $L$  rejects the proposal, and in equilibrium a PEC between  $C$  and  $L$  (i.e.,  $L$ 's preferred option) forms. If instead  $U_{c,lc}^m > U_{c,cr}^m > U_{c,lc}^{\text{pec}}$ ,  $L$  knows that a merger between  $R$  and  $C$  (its least preferred option) would



**Figure 4** – Equilibrium featuring mergers (orange region) and PECs (blue region) as a function of  $\psi$  (x axis) and parties' discount factor  $\delta$  (y axis). Parties' bliss points are set to  $z_l = -0.6, z_c = 0, z_r = 0.7$ . Voters bliss points are uniformly distributed in  $[-1, 1]$ .

form following a rejection. In the latter case,  $L$  accepts  $C$ 's offer and a merger between  $C$  and  $L$  forms.

Finally, when electoral volatility is very high  $\psi < \tilde{\psi}$ , we know from Lemma 3 that party  $C$  prefers to merge with the extreme party  $R$ . We also know from Proposition 2 that party  $R$  always accepts a merger proposal, thus a merger between  $C$  and  $R$  forms when  $\psi < \tilde{\psi}$ .<sup>16</sup>

To summarize, when electoral volatility is low enough (i.e., for  $\psi$  high enough, blue region),  $C$ 's best option is to form a PEC with  $L$  in  $t = 1$ , as this choice ensures the flexibility to form the best alliance in  $t = 2$ . Yet, as electoral volatility increases (i.e., as  $\psi$  decreases), the expected cost of being left out from a coalition becomes more important, and  $C$  prefers to form a merger with the closest party to insure itself against such an outcome (orange region). Finally, when electoral volatility is extremely high, in equilibrium a merger between  $C$  and the more extreme  $R$  could emerge (gray region).

<sup>16</sup>Notice that the boundary between the two merger equilibria regions is not exactly vertical. This happens because a lower discount factor mutes the extent to which less electoral volatility results in a merger equilibrium with the extreme party.



### 3.5. Party Ideological Polarization

How does an increase in ideological polarization affect the equilibrium of the game? Generally, the term ideological polarization might refer to two related, yet distinct, concepts. The first concept concerns the policy positions of different parties. This is the meaning adopted by American politics scholars such as [McCarty, Poole and Rosenthal \(2016\)](#), and by recent Comparative politics literature ([Dalton, 2008](#); [Indridason, 2011](#)). The second concept relates to voters' polarization. In what follows I focus on the first concept of party polarization and see how this impacts the equilibrium party system.<sup>17</sup>

Defining party polarization in multi-party systems is not straightforward, because a good measure requires to take into account both the ideological position of parties as well as their vote share. Intuitively, a highly polarized system is one in which big parties (or coalitions of parties) are located at the opposite extremes of the policy spectrum. The empirical literature on coalition formation has typically operationalized polarization with "ideological division," which represents the greatest ideological distance between any two parties (within the coalition and the opposition). However, as [Indridason \(2011\)](#) notes, this measure does not satisfy some properties expected in a definition of polarization, such as responsiveness to moderate parties' movements.<sup>18</sup>

For the purpose of this model, I will consider the following working definition: polarization increases if the distance of any party from the policy space center increases.<sup>19</sup> The question then is how an increase in polarization, or parties' ideological extremism, affects the sustainability of different alliances in equilibrium.

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<sup>17</sup>While I do not analyze here the concept of voter polarization, it would be interesting to study how different distributions of voters' preferences (e.g., a more extreme electorate) change the supply of parties.

<sup>18</sup>For example, suppose that, all else equal, the central party moves to the right. In this case, polarization should increase because the right becomes more cohesive and the gap between the left and the right increases. Similarly, suppose that the left and the right parties are equidistant from the center party, and that their vote share increases without changing their platforms' location. In this case as well polarization should increase. Yet, the ideological division measure remains constant in both examples.

<sup>19</sup>A limitation of this definition is that by changing the location of parties' platforms, the relative vote share of parties changes as well, because in the model voters are uniformly distributed over  $\mathcal{Z}$ .

The next result assumes that the centrist party lies in the middle of the policy space and studies movements in the location of the right party. An increase in ideological extremism amounts to an increase in  $z_r$ , keeping  $z_l$  fixed.

**Remark 1.** Ideological Extremism. *Let  $z_c = 0$ . Then,*

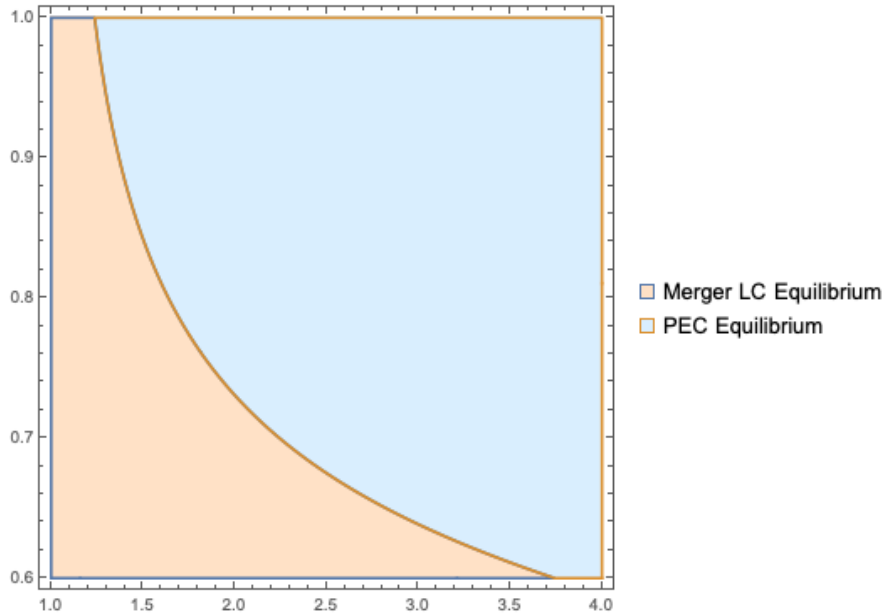
$$\frac{\partial [\partial(U_{c,lc}^m - U_{c,lc}^{pec})/\partial\psi]}{\partial z_r} < 0. \quad (18)$$

From Proposition 2, we know that  $\partial(U_{c,lc}^m - U_{c,lc}^{pec})/\partial\psi < 0$ . That is,  $C$ 's incentives to merge with  $L$  increase with electoral volatility. Remark 1 shows that the magnitude of this incentive varies with  $R$ 's extremism: the negative cross-partial implies that an increase in volatility expands the region of the parameter space supporting a merger equilibrium more when  $R$  is closer to the center of the policy space. **Intuitively, when the extreme party is more of a threat for  $C$ , because it could win an absolute majority by itself, then  $C$  is more prone to form a merger with the closest party  $C$  as a consequence of an increase in volatility.** Conversely, as  $z_r$  moves away from the center, the advantage of merging vis-à-vis forming a PEC shrinks.

Figure 5 provides an illustration of this result, showing how the equilibrium regions vary as a function of  $R$ 's ideological extremism ( $y$  axis) and  $\psi$  ( $x$  axis), for  $z_c = 0$  and  $z_l = -0.6$ . When electoral volatility is low enough ( $\psi$  is high),  $C$  is always better off forming a PEC regardless of the location of  $z_r$  (the more extreme party). As  $\psi$  decreases, the region such that a merger emerges in equilibrium (orange region) expands. In particular, as  $z_r$  becomes more extreme, the merger region becomes less elastic to changes in electoral volatility.

## 4. Alternative Power Sharing Arrangements

How do different configurations of inter-party power sharing affect parties' decision to organize into different types of alliances? The baseline model assumes that the implemented policy coincides with the preferred platform of the party (or coalition) that wins the election: i.e., the party with the majority of votes entirely controls the policy-making process. I refer to this as the *centralized-power model*. However, we might think of policies as a compromise among the policy positions of multiple parties composing the legislature. In consensual



**Figure 5** – Equilibrium featuring mergers (orange region) and PECs (blue region) as a function of  $\psi$  (x axis) and  $z_r$  (y axis). The other parameters are set to  $z_c = 0$ ,  $z_l = -0.6$ ,  $\delta = 0.8$ ,  $a = 1$  and  $\phi = 1/2$ .

democracies, multiple parties typically exercise or have the potential to exercise significant policy influence (Lijphart, 1984).

This section varies the extent to which government policies reflect power-sharing among all parties as opposed to being determined by a single party.

Alternatively to the baseline model — and at the other extreme — Section 4.1 analyzes the case where the implemented policy is a compromise among the policy positions of all the parties composing the parliament, without regard to whether these parties are in government or opposition, weighted by their seat shares. I refer to this specification as the *parliamentary-mean model* of policy (Merrill and Adams, 2007). Proposition 3 below demonstrates that under the parliamentary-mean model no type of pre-electoral alliance is sustainable (neither PECs nor mergers) and in equilibrium parties always run alone.

In reality, implemented policies do not go entirely to one party or coalition, nor are a pure compromise among all parties in the legislature. Section 4.2 takes into account intermediate configurations of institutional power sharing. Whether the policy-making process resembles more the centralized-party model or the parliamentary-mean one depends on country-

specific power sharing arrangements that vary the extent to which power is concentrated or shared with minority parties. The analysis of this unified model shows that pre-electoral alliances (both mergers and PECs) can emerge in equilibrium under intermediate power sharing arrangements.

#### 4.1. Parliamentary-Mean Model

Let the implemented policy be a function of parties' platforms ( $z_i$ ) and their legislative power, measured by seat shares. For simplicity, I assume that parties' seat shares are exactly proportional to vote shares, or in other words that the electoral system is perfectly proportional.<sup>20</sup> Then, the implemented policy function is

$$\hat{x}_t(z_i) = \sum_{i=l,c,r} V_{i,t} \times z_i. \quad (19)$$

This formulation reflects the weight each party has in the post-electoral bargaining process in the legislature. The next result describes the equilibrium of the game under the parliamentary-mean assumption of policy outcomes.

**Proposition 3.** *Parliamentary-Mean Equilibrium. Let the implemented policy be an average of all parties' preferred policies, weighted by parties' vote shares. In equilibrium, neither mergers nor PECs are sustainable, and parties run alone in both periods.*

Proposition 3 shows that institutions that promote compromise and power-sharing among political parties remove parties' need to join pre-electoral alliances to have their platform counted in the implemented policy. In other words, under consensual political institutions there is no premium for the winner of the election in terms of legislative power.

To see why no pre-electoral alliance emerges in equilibrium, let us analyze first parties' decision to form PECs vis-à-vis running alone in the second period. Contrary to the baseline model, where the implemented policy is determined by the winner of the election, the

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<sup>20</sup>The degree to which a PR system resembles perfect proportionality in reality depends on many factors such as district magnitude (i.e. the number of seats awarded per district) and the existence (or absence) of electoral thresholds defined in terms of a minimum percentage of the national vote a party must win in order to guarantee parliamentary representation (Shugart and Taagepera, 1989; Cox, 1997; Lijphart, 2012). Among the most perfectly proportional systems are those of Israel, the Netherlands, and the Scandinavian countries (Lijphart, 2012).

implemented policy under the parliamentary mean model reflects parties' compromise and bargaining taking place after the election. Intuitively, in the second period PECs are always weakly dominated by the choice of running alone because post-electoral negotiations can always reach a policy that is obtained with PECs.

The first period decision is not as trivial as the second period's one because of parties' uncertainty over the electorate's volatility. Because the first period payoff from forming a merger or a PEC is the same,<sup>21</sup> we can focus on parties' comparison of the different continuation values of each alliance configuration. The Appendix shows that there exist a parameter configuration such that  $C$  prefers to merge rather than running alone when electoral volatility is high. This result is due to the concavity of parties' preferences over policies: by merging, the centrist party could prevent a higher policy cost due to one of the extreme parties' policies being weighted more. However, mergers are not incentive compatible for neither  $L$  or  $R$ , which always prefer to run alone for any value of electoral volatility. Thus, in equilibrium no merger forms in the first period and parties compete alone in both periods.

## 4.2. Intermediate Configurations of Power-Sharing

Let  $\alpha \in [0, 1]$  denote the amount of inter-party power sharing in the electoral environment, and consider the following implemented policy:

$$\hat{x}_t(z_i) = \alpha \left( \sum_{i=l,c,r} V_{i,t} \times z_i \right) + (1 - \alpha)z_w, \quad (20)$$

where  $z_w$  is the policy preferred by the party (or coalition) with the plurality of votes. The baseline model assumes that  $\alpha = 0$ , whereas the parliamentary mean model introduced in the previous section assumes that  $\alpha = 1$ . Majoritarian democracies concentrate power in the hands of the winning parties, in such a way that the outcome of the policy making process coincides with the dominant party's preferred policy ( $\alpha = 0$ ). Conversely, in consensual democracies resources are more evenly shared with minority parties, which results in implemented policies partly reflecting the minority's preferences ( $\alpha = 1$ ).

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<sup>21</sup>Recall that the policy resulting from a merger and a PEC between the same parties are equivalent *in the same period*, while leading to different implemented policies in the subsequent period.

Empirically, a change in  $\alpha$  might refer to a change in the electoral system (e.g., from winner-take-all to proportional), or to an institutional change holding fixed the electoral system's proportionality (e.g., from executive dominance to legislative-executive balance). Factors that disperse power among parties in the legislature (increasing  $\alpha$ ) include required supermajorities, bicameral legislatures and provisions for opposition parties' participation on important legislative committees. Factors that promote policy dominance by a single party or by the governing coalition (decreasing  $\alpha$ ) include restrictive legislative procedures (Huber, 1996), unicameral legislatures, and centralized government vis-à-vis federal systems.

We are interested in knowing whether there exists an intermediate level of power sharing that induces parties to form pre-electoral alliances. In other words, is there an  $\alpha \in (0, 1)$  such that either mergers or PECs are sustainable in equilibrium? The subsequent numerical example illustrates parties' incentives under different values of  $\alpha$ .<sup>22</sup>

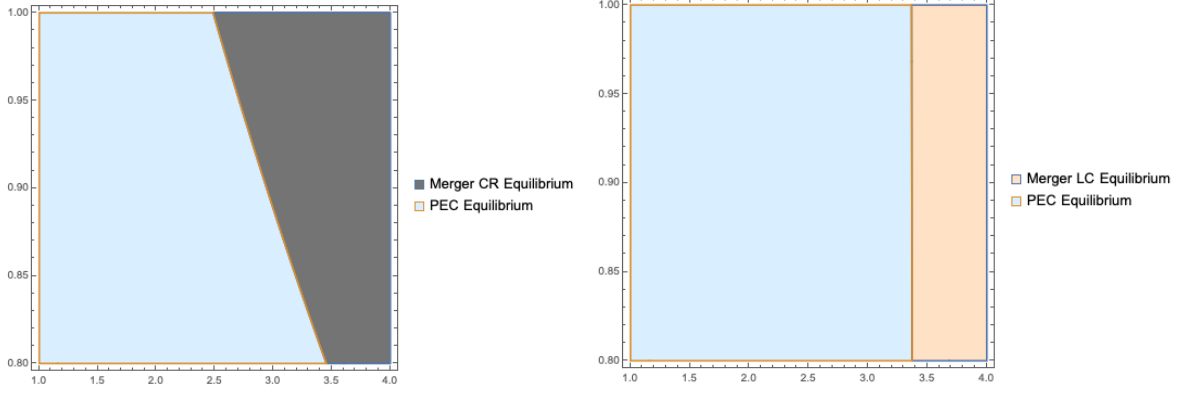
Let  $z_l = -0.6$ ,  $z_c = 0$  and  $z_r = 0.7$ , as in the example in Section 3.4. Figure 6 shows the equilibrium configuration of alliances for different values of  $\alpha$  as a function of electoral volatility. We know from Proposition 3 that for  $\alpha = 1$  no merger is possible in equilibrium, since these are not incentive compatible for either  $L$  or  $R$ . As  $\alpha$  goes down, the implemented policy weighs more the platform of the dominant party. This in turn restores the incentives to merge of  $L$  and  $R$ . In the left panel of Figure 6  $\alpha$  is set to 0.3: for this value, party  $R$  is willing to accept a merger proposal from  $C$ . As  $\alpha$  decreases further, the advantaged party  $L$  is willing to accept  $C$ 's merger proposal: the right panel of Figure 6 shows the equilibrium for  $\alpha = 0.1$ . As the system converges to the dominant model of the baseline, both mergers and PECs are sustainable in equilibrium: when  $\alpha = 0$  the parameter region describing the equilibrium is the one in Figure 4.

## 5. Introducing Uncertainty over Platforms' Location

While each party is associated with a particular policy (its "brand"),  $z_i$ , parties typically feature heterogeneous preferences inside them. This heterogeneity is crucial, as the policy platform that is chosen by each party in a given election might differ from its policy brand

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<sup>22</sup>Unfortunately the complexity of the objective makes an analytic characterization of the equilibrium difficult.



**Figure 6** – Equilibrium type as a function of  $\psi$  (x axis) and parties' discount factor  $\delta$  (y axis), for  $\alpha = 0.3$  (left panel) and  $\alpha = 0.1$  (right panel). Parties' bliss points are set to  $z_l = -0.6$ ,  $z_c = 0$ ,  $z_r = 0.7$ . Voters bliss points are uniformly distributed in  $[-1, 1]$ .

(or, in other words, parties cannot fully pre-commit to policies). This section formalizes this idea by introducing noise in the location of parties' platforms.

Let  $x_{i,t}$  be the policy platform that is selected by party  $i$  in a given election. This platform corresponds to the realization of the random variable  $X_{i,t} = z_i + \epsilon$ , where  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ . The smaller  $\epsilon$ , the sharper the message of the party (i.e., the most informative the party brand). We can interpret the support of  $X_i$  as follows. Parties typically gather multiple candidates who are proponents of different issues, some of which might be very far from the party brand. Depending on which of these candidates wins the election, the party policy could differ from the ex-ante party brand.

When  $L$  and  $C$  merge the resulting policy is a convex combination of the constituent parties' bliss points:

$$X_{lc,1}^m = z_{lc,1}^m + \epsilon^m, \quad (21)$$

where  $\epsilon^m \sim \mathcal{N}(0, \sigma_m^2)$ , and

$$\sigma_m^2 = \sigma^2 + \frac{|z_l - z_c|}{\gamma}. \quad (22)$$

By creating a new political entity, mergers decreases the informativeness of the constituent parties' brands: for any distinct pair of platforms  $z_l$  and  $z_c$ ,  $\sigma_m^2 > \sigma^2$  for any  $\gamma \in \mathbb{R}_+$ . The noise that arises from a merger is increasing in the distance between its constituent parties' bliss points: since voters expect candidates to be drawn from anywhere between  $z_c$  and  $z_l$ ,

the uncertainty cost increases with the distance among platforms.<sup>23</sup> Furthermore, the noise is decreasing in  $\gamma$ : as  $\gamma \rightarrow \infty$ ,  $\sigma_m^2 \rightarrow \sigma^2$ . As such,  $\gamma$  could be interpreted as the amount of trust between the merger's partners.<sup>24</sup> The merged party's brand  $z_{lc,1}^m$  is a convex combination of the constituent parties' bliss points, as in the baseline model (1):  $z_{lc,1}^m = \lambda_{l,1}z_l + (1 - \lambda_{l,1})z_c$ , where  $\lambda_{l,1} = \frac{1}{2} + \phi(V_{l,1} - V_{c,1})$ .

Differently from mergers, PECs preserve the identity of different parties. Thus, when two parties form a PEC the noise term is the same as when parties run individually:  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ .

Because parties cannot pre-commit to policies, voters do not know the exact policy each party selects and suffer an uncertainty cost which is captured by the variance of  $X_i$ . Formally, voter  $v$ 's expected payoff from party  $i$ 's platform is

$$\begin{aligned} EU_v(X_i) &= \mathbb{E}[-(X_i - z_v)^2] \\ &= -(z_i - z_v)^2 - \sigma^2, \end{aligned} \tag{23}$$

where  $z_i = \mathbb{E}[X_i]$  and  $\sigma^2 = \text{Var}[X_i]$ .<sup>25</sup>

To compute each party's vote share when parties run alone, we need to identify the location of the indifferent voter for each pair of parties. Since  $\sigma^2$  is constant across parties, we can focus on the comparison between pairs of party brands ( $l, c$  and  $c, r$ ), as in the baseline model.<sup>26</sup> The same holds when evaluating a PEC's vote share, because of the assumption on the noise term.

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<sup>23</sup>This assumption is supported by empirical evidence showing that mergers are more likely to form between ideologically close parties (Ibenskas, 2016).

<sup>24</sup>When deciding to merge, a party faces the risk that the other partner would renege on the agreement by increasing its policy influence above the agreed at the time of the merger. While I leave it exogenous, it is reasonable to think  $\gamma$  to be positively correlated with the constituent parties' previous experience of governing together, which can reduce the uncertainty about partners' behavior (Franklin and Mackie, 1983; Martin and Stevenson, 2010).

<sup>25</sup>The second equality follows from  $\text{Var}[X_i] = \mathbb{E}[X_i^2] - \mathbb{E}[X_i]^2 = \sigma^2$ , which allows to re-express  $EU_v(X_i)$  as

$$\begin{aligned} EU_v(X_i) &= -\sigma^2 - \mathbb{E}[X_i]^2 + 2\mathbb{E}[X_i]z_v - z_v^2 \\ &= -(\mathbb{E}[X_i]^2 - 2\mathbb{E}[X_i]z_v + z_v^2) - \sigma^2. \end{aligned}$$

<sup>26</sup>For instance, let  $v_{lc,2}$  be the voter who is indifferent between parties  $l$  and  $c$  in  $t = 2$ . Then,  $v_{lc,2}$  solves the same indifference condition as in the baseline model, because the variance terms  $\sigma^2$  cancel out.



The analysis changes when computing the vote share of a merger. Denote by  $v_{lc,r,2}^m$  the voter who is indifferent between party  $R$  and a merger between  $L$  and  $C$  in the second period. That is,  $v_{lc,r,2}^m$  solves:

$$-(v_{lc,r,2}^m - z_{lc,2}^m)^2 - \sigma^2 - \frac{|z_l - z_c|}{\gamma} = -(v_{lc,r,2}^m - z_{r,2})^2 - \sigma^2. \quad (24)$$

From the indifference condition (24) it is clear that parties sacrifice at least some of their vote share when deciding to merge (vis-à-vis forming a PEC). This is because — when  $z_l$  and  $z_c$  differ — voters pay an uncertainty cost when voting for a merged party. Despite this cost from merging, the next result shows that the trade-off identified in Proposition 2 holds, as long as the uncertainty cost associated to the merger is not too high.

**Proposition 4.** *Equilibrium with Electoral Uncertainty. When  $\gamma$  is high enough, in equilibrium parties form mergers when electoral volatility is sufficiently high (low  $\psi$ ), and PECs for low electoral volatility (high  $\psi$ ). When  $\gamma$  is low, in equilibrium  $C$  forms a PEC with the closest party ( $R$ ).*

Intuitively, Proposition 4 shows that mergers are only sustainable if they don't introduce excessive uncertainty about where the party platform stands. This can be the case for example when the merged party has a clear statute which is credible given the constituent parties' histories. Low uncertainty can also be a reasonable assumption if constituent parties have been former allies or have had previous experience of governing together. Conversely, Proposition 4 shows that when voters' uncertainty about the new political party is high, a merger is not a viable alternative to a PEC *even when the electorate is very volatile*.

## 6. Conclusion

The majority of multi-party systems are extremely “liquid” (Powell Jr, 2000; Golder, 2006): parties split, merge, form and leave coalitions at all times, and these movements largely affect parties' electoral chances. While the literature typically assumes that each party is associated to a particular policy platform — highlighting the important role of parties in producing political brand names (Downs, 1957; Snyder and Ting, 2002) —, in multi-party systems each party is often associated to different brands depending on the allies chosen.

Thus, in the context of multi-party competition it is unclear “who owns the party brand,” given the different alliances parties can form. This is an unfortunate gap because understanding the different forms of inter-party cooperation is crucial for anticipating the development of party systems. To fill such void, this paper presents a simple model of electoral competition in which parties form alliances before elections, and decide how binding these alliances should be.

The central intuition of the model is that parties’ strategic choice of electoral alliances crucially depends on the underlying volatility of the electorate. In particular, Proposition 2 suggests that stable electorates might incentivize flexible types of coalitions that are renegotiated in every election. Conversely, unstable electorates might be empirically correlated with more binding alliances such as mergers. Recent political developments have brought attention to the electoral decline of established parties and the burst of electoral volatility following the Great Recession of 2007 in Europe. The result suggests that this increased electoral volatility might lead to an increase in the number of binding coalitions in the future.

The model produces several empirical implications. Results suggest that we should expect mergers to be empirically associated with volatile electorates. In principle, an accurate measure of electoral volatility should reflect the extent to which personal votes change between subsequent elections. Thus, individual level data identifying voters’ intentions to vote or party identification across time represent an accurate measure of volatility.

Results also show that at least some degree of power concentration is needed to trigger mergers and pre-electoral coalitions. Under consensual democracies that share power among all parties, minority parties do not need to join pre-electoral alliances to have their voices heard in the policy-making process. As power gets more concentrated in the hands of the winner of the election, parties need to join forces and both PECs and mergers can emerge in equilibrium. A decrease in power sharing might refer to a change in the electoral system (e.g., from proportional to winner-take-all), or to an institutional change holding fixed the electoral system’s proportionality (e.g., from legislative-executive balance to executive dominance).

While identifying future governments in two-party systems such as the United States is straightforward, it is unusual for almost every other democracy for a single party to win the

majority of votes, making the identity of government more uncertain. By making explicit the identity of future governments, pre-electoral alliances (both mergers and PECs) have the important role of creating mandate conditions in multi-party systems.

These findings contribute to our understanding of party systems. Binding alliances such as mergers can reduce excessive party system fragmentation by forming stable parties. In the short term, however, mergers can reduce the information value of party labels for voters thus hindering voter representation and accountability. Ultimately, understanding the outcomes of party system formation and stabilization is not possible without considering the role of mergers and pre-electoral coalitions. While this paper only begins to unpack the incentives behind different forms of pre-electoral alliances, future research should further investigate how these incentives change with alternative institutional and non-institutional features of the competitive environment in which parties operate.

## 7. Appendix

**Proof of Lemma 1.** Let  $\Delta_c^{\text{pec}}(\xi) = u_c(z_{lc,2}^{\text{pec}}) - u_c(z_{cr,2}^{\text{pec}})$ , where

$$\Delta_c^{\text{pec}}(\xi) = \frac{(z_c - z_r)^2[\phi(z_c - z_r - 2(\xi + z_r)) - 2a(\phi + 1)]^2 - (z_c - z_l)^2[2a(\phi + 1) + \phi(z_c - z_r + 2(z_l - \xi))]^2}{16a^2}.$$

Differentiating  $\Delta_c$  with respect to  $\xi$  yields

$$\frac{(1 + \phi)(2z_c^2 - 2z_c(z_l + z_r) + z_l^2 + z_r^2) + \phi(z_l - z_r)(z_c^2 + z_c(4\xi - 3(z_l + z_r))) + 2z_l^2 + z_l(z_r - 2\xi) + z_r(z_r - \xi)}{4a^2},$$

which is always negative. □

**Proof of Lemma 2.** Differentiating  $\Delta_c$  with respect to  $z_r$  yields

$$\frac{a(\phi + 1)\phi(z_c^2 + z_c(-4\xi - 4z_l + 6z_r) - 2a^2(\phi + 1)^2(z_c - z_r) + z_l^2 + 2z_lz_r - 6z_r^2 + 4\xi z_r) + z_c^3 - 2z_cz_l^2 + \phi^2(-\xi(z_c^2 - 2z_r(3z_c + z_l) + z_l^2 + 6z_r^2) - 2z_c^2z_r + 4z_cz_lz_r - 3z_cz_r^2 + 2\xi^2(z_r - z_c) + z_l^3 - 3z_lz_r^2 + 4z_r^3)}{4a^2},$$

which is always positive. □

**Proof of Lemma 3.** Let  $\Delta_c^m(\psi) = U_{i,lc}^m - U_{i,cr}^m$ . For ease of exposition, let  $a = 1$  and  $z_c = 0$ .<sup>27</sup>

Then:

$$\begin{aligned} \Delta_c^m(\psi) &= \delta z_l^2 (\psi z_l(z_r\phi + 4) - 2\psi z_r^2\phi + 2\psi z_r(\phi + 1) + 8) + z_r^2(\phi(z_l - 2z_r + 2) + 2)^2 \\ &\quad - \frac{1}{4}\delta z_l^2(\phi(2z_l - z_r + 2) + 2)^2 \left[ \psi \left( \frac{1}{4}z_l(\phi(2z_l - z_r + 2) + 2) + z_r \right) + 2 \right] \\ &\quad + z_l^2(-(\phi(2z_l - z_r + 2) + 2)^2) + 4\delta z_r^2 \left[ \psi \left( \frac{1}{4}z_l(\phi(2z_l - z_r + 2) + 2) + z_r \right) - 2 \right] \\ &\quad + \delta z_r^2(\phi(z_l - 2z_r + 2) + 2)^2 \left[ \frac{1}{2} - \frac{1}{4}\psi \left( \frac{1}{4}z_r(\phi(z_l - 2z_r + 2) + 2) + z_l \right) \right]. \end{aligned}$$

Differentiating  $\Delta_c^m(\psi)$  with respect to  $\psi$  yields

$$-\frac{1}{4}z_l^3(\phi(2z_l - z_r + 2) + 2)^3 - z_l^2z_r(\phi(2z_l - z_r + 2) + 2)^2 + 4z_l^2z_r(\phi(z_l - 2z_r + 2) + 2)$$

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<sup>27</sup>The result does not depend on  $C$  being located in the middle of the policy space. The expressions for general  $z_c$  are available upon request.

$$+ 16z_l^3 - z_l z_r^2 (\phi(z_l - 2z_r + 2) + 2)^2 + 4z_l z_r^2 (\phi(2z_l - z_r + 2) + 2) - \frac{1}{4} z_r^3 (\phi(z_l - 2z_r + 2) + 2)^3 + 16z_r^3$$

which is always negative.  $\square$

**Proof of Lemma 4.** Differentiating  $\Delta_c^m(\psi)$  with respect to  $z_r$  yields

$$\begin{aligned} & 2z_l^2 (4\delta\psi + 3\delta\psi z_l (z_l + 1)^2 \phi^3 + 2(z_l + 1)\phi^2 (\delta(\psi(z_l - 2) + 4) + 8) + \phi(\delta(8 - \psi z_l) + 16)) \\ & - 3z_r^2 (-28\delta\psi + \delta\psi (3z_l^2 + 6z_l + 4) \phi^3 + \phi^2 (4\delta(3\psi + 8) - 3\delta\psi z_l^2 - 4z_l(\delta(\psi - 4) - 8) + 64) \\ & + \phi(\delta(32 - 2\psi(z_l - 6)) + 64)) + z_r (2\delta(-3\psi z_l^3 (z_l + 1)\phi^3 + \phi^2(3\psi z_l^3 - 8(\psi - 2)z_l \\ & + 16) + 8(\psi z_l - 6) + 16(z_l + 2)\phi) + 64(\phi + 1)(z_l \phi + \phi + 1)) + 24\delta\psi z_r^5 \phi^3 \\ & - 30\delta\psi z_r^4 \phi^2 ((z_l + 2)\phi + 2) + 4z_r^3 \phi (12\delta\psi + 3\delta\psi (z_l + 2)^2 \phi^2 + 4\phi(\delta(\psi(z_l + 6) + 4) + 8)), \end{aligned}$$

which is always positive.  $\square$

**Proof of Proposition 2.** The proof proceeds as follows. First, I show that the difference  $U_{c,lc}^m - U_{c,lc}^{\text{pec}}$  is *decreasing* in  $\psi$ : that is, as volatility decreases ( $\psi$  increases),  $C$  prefers a coalition to a merger with the closest party  $L$ .

Next, I consider the following equilibrium candidate:  $C$  proposes a PEC to  $L$  for  $\psi > \hat{\psi}$ , where  $\hat{\psi}$  solves  $U_{c,lc}^m = U_{c,lc}^{\text{pec}}$ , and show that this is incentive compatible for  $L$ . Then, I verify that a merger with  $L$  is incentive-compatible for  $L$  when  $\psi < \hat{\psi}$ .

Finally, I derive  $\tilde{\psi}$ , defined as the value of  $\psi$  such that  $U_{c,cr}^m = U_{c,lc}^m$ . Because of Lemma 3, we have  $U_{c,cr}^m > U_{c,lc}^m$  for  $\psi < \tilde{\psi}$ . Then, I verify that a merger is incentive-compatible for  $R$  for this range of electoral volatility.

For ease of exposition, let  $a = 1$  and  $z_c$  be located at the center of the policy space:  $z_c = 0$  (these assumption only simplify the following expressions but are without loss of generality). From the expression of  $U_{c,lc}^m$  (14), it is straightforward to compute the following derivative:

$$\begin{aligned} \frac{\partial U_{c,lc}^m}{\partial \psi} &= \frac{1}{4} (z_{lc,1}^m + z_r) [u_c(z_{lc,1}^m) - u_c(z_r)] \\ &= \frac{1}{4} \left[ \frac{1}{4} z_l (\phi(2z_l - z_r + 2) + 2) + z_r \right] \left[ z_r^2 - \frac{1}{16} z_l^2 (\phi(2z_l - z_r + 2) + 2)^2 \right]. \end{aligned} \quad (25)$$

Subtracting  $\partial U_{c,lc}^{\text{pec}}/\partial\psi$  from Equation 25 produces

$$\begin{aligned}
& \frac{8\phi(4z_l^2(z_l+1) + (4-3z_l)z_r^2 + z_l(3z_l+8)z_r - 4z_r^3)}{\psi^2(z_l\phi+4)(z_r\phi-4)} \\
& + \frac{4\phi(z_l^2z_r^2\phi^3(-3z_l+3z_r-4) + 4z_lz_r\phi^2(z_l^2+z_l(2-3z_r)+(z_r-2)z_r) + 32(z_l+z_r)^2)}{\psi^2(z_l\phi+4)(z_r\phi-4)} \quad (26) \\
& + \frac{3z_l^2(\phi(2z_l-z_r+2)+2)^2(z_lz_r\phi^2(3z_l-3z_r+4) + 8(z_l+z_r) + 8z_l(z_l+1)\phi - 4(z_r-2)z_r\phi)}{\phi(z_l-z_r)(z_l\phi+4)} \\
& + \frac{48z_l^2(z_r\phi(z_l-2z_r+2) + 2(2z_l+z_r))}{z_r\phi-4} + \frac{48z_r^2(z_l(\phi(2z_l-z_r+2)+2) + 4z_r)}{z_l\phi+4} - \frac{16z_r^2\phi^2}{\psi^3} \\
& + \frac{3z_r^2(\phi(z_l-2z_r+2)+2)^2(z_lz_r\phi^2(-3z_l+3z_r-4) + 8(z_l+z_r) + 4z_l(z_l+2)\phi - 8(z_r-1)z_r\phi)}{\phi(z_l-z_r)(z_r\phi-4)},
\end{aligned}$$

which under the assumptions is always negative.

Let  $\hat{\psi}$  be the value of  $\psi$  such that  $U_{c,lc}^m = U_{c,lc}^{\text{pec}}$  (the expression for  $\hat{\psi}$  is lengthy therefore omitted). It follows from Equation 26 that for  $\psi > \hat{\psi}$ ,  $U_{c,lc}^{\text{pec}} > U_{c,lc}^m$ . Suppose that for this range of volatility  $C$  proposes a PEC to  $L$ .  $L$  accepts because  $U_{l,lc}^{\text{pec}} > U_{l,lc}^m$  for, since  $L$  is closer to  $C$  than  $R$  is and has an electoral advantage. Hence, for  $\psi > \hat{\psi}$ , in equilibrium a PEC between  $C$  and  $L$  forms in  $t = 1$ .

For  $\psi < \hat{\psi}$ ,  $C$  prefers to form a merger with  $L$ . Suppose that  $C$  proposes a PEC to  $L$ . If  $L$  accepts, its expected payoff is  $U_{l,lc}^m$ . If  $L$  rejects, the outcome depends on  $C$ 's ranking of alternatives, which varies with  $\psi$ . That is, if  $U_{c,lc}^m > U_{c,lc}^{\text{pec}} > U_{c,cr}^m$ , knowing  $C$ 's ranking  $L$  rejects the proposal, and in equilibrium a merger between  $C$  and  $L$  (i.e.,  $L$ 's preferred option) forms. If instead  $U_{c,lc}^m > U_{c,cr}^m > U_{c,lc}^{\text{pec}}$ ,  $L$  knows that a merger between  $R$  and  $C$  (its least preferred option) would form following a rejection. In the latter case,  $L$  accepts  $C$ 's offer and a merger between  $C$  and  $L$  forms.

We are left to check whether a merger between  $C$  and  $R$  can form for some  $\psi$ . Let  $\tilde{\psi}$  be the value of  $\psi$  such that  $U_{c,lc}^m = U_{c,cr}^m$ . Solving for  $\psi$  produces

$$\tilde{\psi} = \frac{(z_l^2 + \delta z_l^2(\phi(2z_l - z_r + 2) + 2 - 16\delta))^2 + 2z_l^2(\phi(2z_l - z_r + 2) + 2)^2 - \delta z_r^2(\phi(z_l - 2z_r + 2) + 2)^2 - 2z_r^2(\phi(z_l - 2z_r + 2) + 2)^2 + 16\delta z_r^2}{\delta(4z_l^3(z_r\phi + 4) - 8z_l^2z_r^2\phi + 8z_l^2z_r(\phi + 1) - z_l^2(\phi(2z_l - z_r + 2) + 2)^2(\frac{1}{4}z_l(\phi(2z_l - z_r + 2) + 2) + z_r) + 16z_r^2(\frac{1}{4}z_l(\phi(2z_l - z_r + 2) + 2) + z_r) - z_r^2(\phi(z_l - 2z_r + 2) + 2)^2(\frac{1}{4}z_r(\phi(z_l - 2z_r + 2) + 2) + z_l))}.$$

It follows from Lemma 3 that  $U_{c,cr}^m > U_{c,lc}^m$  for  $\psi < \tilde{\psi}$ .

Finally, let us analyze the incentive compatibility constraint of party  $R$ . It is easy to verify that  $R$  always accept a merger proposal from  $C$ , since the difference We can express the difference as

$$\begin{aligned} U_{r,cr}^m - U_{r,lc}^m &= \frac{z_r^2(\phi(z_l - 2z_r + 2) - 2)^2(\psi z_l(z_r\phi + 4) + 2\psi z_r(-z_r\phi + \phi + 1) - 8)}{256} - \frac{z_r^2(\phi(z_l - 2z_r + 2) - 2)^2}{16} \\ &+ \frac{\delta(z_l(\phi(2z_l - z_r + 2) + 2) - 4z_r)^2(\psi z_l(\phi(2z_l - z_r + 2) + 2 + 4\psi z_r + 8))}{256} \\ &- \frac{1}{4}(z_l - z_r)^2 \left[ \psi \left( \frac{1}{4}z_r(\phi(z_l - 2z_r + 2) + 2) + z_l \right) + 2 \right] + \left( \frac{1}{4}z_l(\phi(-2z_l + z_r - 2) - 2) + z_r \right)^2 \end{aligned}$$

which under the assumptions is always positive. This in turn implies, from the previous step of the proof, that in equilibrium a merger between  $C$  and  $R$  forms for  $\psi < \tilde{\psi}$  and a PEC between  $C$  and  $L$  forms for  $\psi > \hat{\psi}$ . When  $\tilde{\psi} < \psi < \hat{\psi}$ , a merger between  $C$  and  $L$  forms when  $U_{c,lc}^m > U_{c,lc}^{\text{pec}} > U_{c,cr}^m$ , and a PEC between  $C$  and  $L$  forms when  $U_{c,lc}^m > U_{c,cr}^m > U_{c,lc}^{\text{pec}}$ , which completes the proof.  $\square$

**Proof of Remark 1.** Let  $z_r = -z_l + \kappa$ , where  $\kappa > 0$ . The proof simply follows from differentiating Equation 25 with respect to  $\kappa$ , which is always negative for  $\kappa > 0$ .  $\square$

**Proof of Proposition 3.** Denote by  $U_i(-m_1)$  party  $i$ 's second period expected payoff if parties do not merge in  $t = 1$ . Because of the parliamentary-mean assumption over the implemented policy, this is, for party  $C$ :

$$\begin{aligned} U_c(-m_1) &= \int_{-1/\psi}^{1/\psi} u_c(V_{l,2}z_l + V_{c,2}z_c + V_{r,2}z_r) \frac{\psi}{2} d\xi \\ &= -\frac{3(2a(-2z_c + z_l + z_r) + z_l^2 - z_r^2)^2}{48a^2} + \frac{(z_l - z_r)^2}{12(a\psi)^2}, \end{aligned} \quad (27)$$

and analogously for parties  $L$  and  $R$ . Party  $C$  compares  $U_c(-m_1)$  with  $U_{c,2}(m_{lc,1})$  and  $U_{c,2}(m_{cr,1})$  in the first period, when deciding whether to propose a merger to any party.

Let  $\Delta_c^{lc,2} = U_{c,2}(m_{lc,1}) - U_c(-m_1)$ . Differentiating with respect to  $\psi$  yields:

$$\frac{\partial \Delta_c^{lc,2}}{\partial \psi} = \frac{(z_c - z_l)(-4a^2(\phi - 1)(z_c(-\phi) + z_c + z_l(\phi + 3) - 4z_r) + 4a\phi(z_c + 2z_l - z_r)(z_c(\phi - 1) - z_l(\phi + 1) + 2z_r) + \phi^2(z_c - z_l)(z_c + 2z_l - z_r)^2)}{96a^4\psi^3},$$

which is always negative under the assumptions. Furthermore, there exists a value  $\hat{\psi}^{lc}$  such that  $\Delta_c^{lc,2}(\hat{\psi}^{lc}) = 0$  (the expression is long therefore omitted). Hence,  $C$  prefers to merge with  $L$  for  $\psi < \hat{\psi}^{lc}$ , while it prefers the continuation value from a PEC for  $\psi > \hat{\psi}^{lc}$ .

Analogously, we have that  $\partial \Delta_c^{cr,2} / \partial \psi < 0$  and that there exists  $\hat{\psi}^{cr}$  such that  $\Delta_c^{cr,2}(\hat{\psi}^{cr}) = 0$ . Hence,  $C$  prefers to merge with  $R$  for  $\psi < \hat{\psi}^{cr}$ , while it prefers the continuation value from a PEC for  $\psi > \hat{\psi}^{cr}$ .

It is left to show that a merger is not incentive compatible neither for  $L$  nor for  $R$ . For  $L$ , we have that

$$\begin{aligned} \Delta_l^{lc,2} &= \frac{1}{48}(z_l - z_r)^2 (3z_l^2 + 6z_l(z_r - 2) + 3z_r^2 - 12z_r + 16) \\ &- \frac{1}{(\phi(z_c - z_l)(z_c + 2z_l - z_r) - 2(z_c(-\phi) + z_c + z_l\phi + z_l - 2z_r))} \left[ ((\phi(z_c - z_l)(z_c + 2z_l - z_r) - 2(z_c(-\phi) \right. \\ &+ z_c + z_l\phi + z_l))(-(\phi(z_c - z_l)(z_c + 2z_l - z_r)) + 2(z_c(-\phi) + z_c + z_l\phi + z_l + 2z_r + 4) + 8) \\ &- 4z_r(\phi(z_c - z_l)(z_c + 2z_l - z_r) - 2((1 - \phi)z_c + (1 + \phi)z_l + 2z_r) + 8) + 64z_l)^3 - ((\phi(z_c - z_l)(z_c + 2z_l - z_r) \\ &- 2((1 - \phi)z_c + z_l\phi + z_l))(-(\phi(z_c - z_l)(z_c + 2z_l - z_r)) - 2((1 - \phi)z_c + (1 + \phi)z_l + 2z_r - 4) + 8) + 64z_l)^3 \\ &\left. + 2(z_c(-\phi) + z_c + z_l\phi + z_l + 2z_r - 4) + 8) - 4z_r(\phi(z_c - z_l)(z_c + 2z_l - z_r)) \right], \end{aligned}$$

which is always negative under the assumptions. It follows that  $L$  rejects a merger proposal by  $C$ . Similarly,  $\Delta_r^{cr,2} < 0$ , and  $R$  rejects a merger proposal by  $C$ .

Since mergers are always dominated for both  $L$  and  $R$ , in equilibrium no alliance forms in  $t = 1$ , and the unique equilibrium for all parameter values is that all parties run alone.  $\square$

**Proof of Proposition 4.** The analysis of  $t = 2$  is analogous to the baseline model. First, suppose that no merger formed in  $t = 1$ . Because  $\sigma_m^2 > \sigma^2$ , mergers are dominated in the second period, and both voters' and parties' decision are identical to the baseline model.



Suppose instead that a merger between  $C$  and  $R$  formed in  $t = 1$ . By assumption, the merger persists and faces party  $L$ . Notice that the probability that the merged party gets the majority in  $t = 2$  is  $\Pr\{\xi > \tilde{\xi}_r\} = 1 - F(\tilde{\xi}_r)$  (the same as in the baseline), because the informational cost is only paid by voters in  $t = 1$  when the merger is formed. Hence, the expected second period payoff from merging (14) is the same as in the baseline model.

In  $t = 1$ , policy uncertainty introduced by mergers changes how vote shares are computed. Let  $v_{l,cr,2}^m$  denote the voter who is indifferent between voting for party  $L$  and for a merger among  $C$  and  $R$ . Formally,  $v_{l,cr,2}^m$  solves

$$-(v_{l,cr,2}^m - z_{cr,2}^m)^2 - \sigma^2 - \frac{z_c - z_r}{\gamma} = -(v_{l,cr,2}^m - z_{l,2})^2 - \sigma^2. \quad (28)$$

Solving for the indifferent voter yields:

$$v_{l,cr,2}^m = -\frac{4a^2(\gamma z_c^2(\phi - 1)^2 - 2z_c(\gamma z_r(\phi^2 - 1) + 2) - 4\gamma z_l^2 + \gamma z_r^2(\phi + 1)^2 + 4z_r) - 4\gamma a\phi(z_c - z_r)(z_c - z_l + 2z_r)(z_c(\phi - 1) - z_r(\phi + 1)) + \gamma\phi^2(z_c - z_r)^2(z_c - z_l + 2z_r)^2}{8a\gamma(2a(z_c(\phi - 1) + 2z_l - z_r(\phi + 1)) - \phi(z_c - z_r)(z_c - z_l + 2z_r))}. \quad (29)$$

Using this expression, it is straightforward to compute the vote share of the merged party in  $t = 1$ :

$$V_{cr,1}^m = \frac{1}{2} + \frac{(z_r - z_c)}{\gamma(\phi(z_c - z_r)(2a - z_c + z_l - 2z_r) - 2a(z_c - 2z_l + z_r))} - \frac{z_c + 2z_l + z_r}{8a} + \frac{\phi(z_c - z_r)(2a - z_c + z_l - 2z_r)}{16a^2}$$

Differentiating  $V_{cr,1}^m$  with respect to  $\gamma$  yields

$$\frac{z_c - z_r}{\gamma^2(2a(z_c(\phi - 1) + 2z_l - z_r(\phi + 1)) - \phi(z_c - z_r)(z_c - z_l + 2z_r))}, \quad (30)$$

which is always positive: as  $\gamma$  increases, the uncertainty paid by voter is reduced and the vote share of the merger increases.

Finally, we check if there exists a positive  $\gamma$  such that  $V_{cr,1}^m = 1/2$ . Solving for  $\gamma$  yields

$$\hat{\gamma} = \frac{16a^2(z_c - z_r)}{4a^2((z_c + z_r)^2 - 4z_l^2) + \phi^2(z_c - z_r)^2(2a - z_c + z_l - 2z_r)^2 - 4a\phi(z_c^2 - z_r^2)(2a - z_c + z_l - 2z_r)}, \quad (31)$$

which is a positive real root. It follows that for  $\gamma > \hat{\gamma}$ ,  $V_{cr,1}^m > 1/2$  and the analysis is analogous to the proof of Proposition 2. In particular, let  $\Delta_{c,cr} \equiv U_{c,cr}^m - U_{c,lc}^{\text{pec}}$ , where

$$U_{c,cr}^m = -(z_{cr,1}^m - z_c)^2 - \sigma^2 - \frac{|z_c - z_r|}{\gamma} + \delta U_{i,2}(m_{cr}),$$

and

$$U_{c,lc}^{\text{pec}} = -(z_{lc,1}^{\text{pec}} - z_c)^2 - \sigma^2 + \delta U_{i,2}(-m).$$

Because uncertainty only affects  $\Delta_{c,cr}$  via the term  $\frac{|z_c - z_r|}{\gamma}$ , it follows that  $\partial(U_{c,cr}^m - U_{c,lc}^{\text{pec}})/\partial\psi$  is always negative, analogously to Equation 26. Furthermore, for  $\gamma$  big enough, there exists a value of  $\psi$  such that  $U_{c,cr}^m = U_{c,lc}^{\text{pec}}$ , and the result in Proposition 2 continues to hold.

It is left to show that for  $\gamma$  small enough no mergers are sustainable in equilibrium. When  $\gamma < \hat{\gamma}$ ,  $V_{cr,1}^m < 1/2$ . In this case we have

$$U_{c,cr}^m = -(z_{cr,1}^m - z_c)^2 - \sigma^2 - \frac{|z_c - z_r|}{\gamma} + \delta U_{i,2}(m_{cr}),$$

Notice that  $U_{i,cr}^m \rightarrow -\infty$  as  $\gamma \rightarrow 0$ . This implies that there exists  $\gamma'$  small enough such that  $\Delta_{c,cr}(\gamma') = 0$  has no solution. In particular, we have  $U_{c,lc}^{\text{pec}}(\gamma') > U_{i,cr}^m(\gamma')$  for all  $\psi$ . The analysis for a merger between  $C$  and  $L$  is analogous therefore omitted.  $\square$

## References

- Bartolini, Stefano and Peter Mair. 1990. *Identity, Competition, and Electoral Availability: The Stabilisation of European Electorates 1885-1985*. Cambridge: Cambridge University Press.
- Bélanger, Éric and Jean-François Godbout. 2010. "Why do Parties Merge? The Case of the Conservative Party of Canada." *Parliamentary Affairs* 63(1):41–65.
- Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge University Press.
- Dalton, Russell J. 2008. "The Quantity and the Quality of Party Systems: Party System Polarization, Its Measurement, and Its Consequences." *Comparative Political Studies* 41(7):899–920.
- Downs, Anthony. 1957. *An Economic Theory of Democracy*. New York; Harper and Row.
- Emanuele, Vincenzo. 2015. "Dataset of Electoral Volatility and its Internal Components in Western Europe (1945-2015)." *Rome: Italian Center for Electoral Studies* .
- Franklin, Mark N and Thomas T Mackie. 1983. "Familiarity and Inertia in the Formation of Governing Coalitions in Parliamentary Democracies." *British Journal of Political Science* 13(3):275–298.
- Frey, Anderson, Gabriel López-Moctezuma and Sergio Montero. 2021. "Sleeping with the Enemy: Effective Representation under Dynamic Electoral Competition." *Unpublished Manuscript* .
- Golder, Sona Nadenichek. 2006. *The Logic of Pre-Electoral Coalition Formation*. Ohio State University Press.
- Hortala-Vallve, Rafael, Jaakko Meriläinen and Janne Tukiainen. 2021. "Pre-Electoral Coalitions: Insights into the Creation of Political Parties." *Unpublished Manuscript* .
- Huber, John D. 1996. "The Vote of Confidence in Parliamentary Democracies." *American Political Science Review* pp. 269–282.

- Hug, Simon. 2001. *Altering Party Systems: Strategic Behavior and the Emergence of New Political Parties in Western Democracies*. University of Michigan Press.
- Ibenskas, Raimondas. 2016. "Marriages of Convenience: Explaining Party Mergers in Europe." *The Journal of Politics* 78(2):343–356.
- Ibenskas, Raimondas and Allan Sikk. 2017. "Patterns of Party Change in Central and Eastern Europe, 1990–2015." *Party Politics* 23(1):43–54.
- Ibenskas, Raimondas and Nicole Bolleyer. 2018. "Forms of inter-party cooperation: Electoral coalitions and party mergers." *East European Politics and Societies* 32(3):451–472.
- Indridason, Indridi H. 2011. "Coalition formation and polarisation." *European Journal of Political Research* 50(5):689–718.
- Kitschelt, Herbert, Zdenka Mansfeldova, Radoslaw Markowski and Gabor Toka. 1999. *Post-Communist Party Systems: Competition, Representation, and Inter-Party Cooperation*. Cambridge University Press.
- Koole, Ruud. 1994. "The Vulnerability of the Modern Cadre Party in the Netherlands." *How Parties Organize: Change and Adaptation in Party Organizations in Western Democracies* pp. 278–303.
- Lijphart, Arend. 1984. *Democracies: Patterns of Majoritarian and Consensus Government in Twenty-One Countries*. Yale University Press New Haven.
- Lijphart, Arend. 2012. *Patterns of Democracy: Government Forms and Performance in Thirty-Six Countries*. Yale University Press.
- Martin, Lanny W and Randolph T Stevenson. 2010. "The Conditional Impact of Incumbency on Government Formation." *American Political Science Review* pp. 503–518.
- McCarty, Nolan, Keith T Poole and Howard Rosenthal. 2016. *Polarized America: The Dance of Ideology and Unequal Riches*. MIT Press.

- Merrill, Samuel and James Adams. 2007. "The Effects of Alternative Power-sharing Arrangements: Do "Moderating" Institutions Moderate Party Strategies and Government Policy Outputs?" *Public Choice* 131(3):413–434.
- Mershon, Carol and Olga Shvetsova. 2013. "The Microfoundations of Party System Stability in Legislatures." *The Journal of Politics* 75(4):865–878.
- Olsen, Jonathan. 2007. "The Merger of the PDS and WASG: From Eastern German Regional Party to National Radical Left Party?" *German Politics* 16(2):205–221.
- Pedersen, Mogens N. 1979. "The Dynamics of European Party Systems: Changing Patterns of Electoral Volatility." *European Journal of Political Research* 7(1):1–26.
- Powell Jr, G Bingham. 2000. *Elections as Instruments of Democracy: Majoritarian and Proportional Visions*. Yale University Press.
- Rakner, Lise, Lars Svåsand and Nixon S Khembo. 2007. "Fissions and Fusions, Foes and Friends: Party System Restructuring in Malawi in the 2004 General Elections." *Comparative Political Studies* 40(9):1112–1137.
- Shugart, Matthew Soberg and Rein Taagepera. 1989. *Seats and Votes*. New Haven.
- Sikk, Allan. 2005. "How unstable? Volatility and the Genuinely New Parties in Eastern Europe." *European Journal of Political Research* 44(3):391–412.
- Snyder, James M and Michael M Ting. 2002. "An Informational Rationale for Political Parties." *American Journal of Political Science* pp. 90–110.
- Tavits, Margit. 2008. "On the Linkage Between Electoral Volatility and Party System Instability in Central and Eastern Europe." *European Journal of Political Research* 47(5):537–555.