Electoral Volatility and Pre-Electoral Alliances

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Abstract

In multi-party systems, parties often form alliances before elections. What brings competing parties to coalesce into new entities? I present a model of electoral competition in which parties can form pre-electoral alliances and decide how binding these should be. Parties face a dynamic trade-off between insuring themselves against significant shifts in public opinion and allowing flexibility to respond to future electoral changes. The model shows that more binding alliances such as mergers emerge in equilibrium when electoral volatility is high; instead, when voters are predictable (e.g., highly partisan), parties either run alone or form more flexible pre-electoral coalitions. When the electorate is sufficiently volatile, a risk-averse centrist party might prefer an extreme merger partner to a moderate one. Furthermore, alliances only emerge under some power concentration, whereas parties run alone under consensual democracies that share power among winners and losers.

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1. Introduction

In multi-party systems, parties frequently form alliances before elections. A common way for different parties to form an alliance is to support joint candidates, while competing with their separate identities in other areas (e.g., member recruitment). This form of alliance, typically referred to as a *pre-electoral coalition*, is often chosen by parties to join forces against strong opponent candidates. For example, recent evidence from Mexican and Finnish local elections demonstrates that parties are willing to form pre-electoral alliances to remove entrenched incumbent parties from office (Frey, López-Moctezuma and Montero, 2021; Hortala-Vallve, Meriläinen and Tukiainen, 2021).

A common alternative to pre-electoral coalitions used by parties that want to join forces is to *merge* into a new single-party organization. In Europe, mergers have occurred on average every third electoral period since World War II.[1] Political leaders often consider the option of merging when facing electoral uncertainty, as the UK case illustrates. Opinion polls in 1986 indicated that all major parties led at various points,[2] and in 1988 the SPD and the Liberal Party formally merged as the Social and Liberal Democratic Party. In France, a volatile electorate allowed the far-right candidate Jean Marie Le Pen to reach the second round of the 2002 presidential election. The event triggered turbulence, deep political upheaval and institutional change.[3] In response, The UMP was formed in September 2002 as a merger of several centre-right parties under the leadership of President Jacques Chirac.

Unlike pre-electoral coalitions, which tend to be temporary, mergers lead to significant changes in the party system. The Italian political landscape completely changed in 2007, when mergers across the ideological spectrum effectively transformed the system into bipolarism, with two main competing electoral cartels. Pre-electoral alliances such as mergers facilitate

[1] Ibenskas (2016) collected a dataset that considers 280 democratic elections in the postwar period in European countries. Overall, the dataset includes 94 mergers formed by 216 parties. These mergers occurred over 59 electoral periods and were predominantly formed by two parties.

[2] Cf. Hughes, D. (2019) 'Volatile voting: Why the next General Election is going to be a shambles', Electoral Reform Society: https://www.electoral-reform.org.uk/volatile-voting-why-the-next-general-election-is-going-to-be-a-shambles/.

[3] Cf. Daley, S. (2002) 'THE FRENCH SURPRISE: THE SHOCK; French Political Leaders Rally Around Chirac', New York Times, 23 April: https://www.nytimes.com/2002/04/23/world/the-french-surprise-the-shock-french-political-leaders-rally-around-chirac.html..

the formation of durable parties and can reduce party system fragmentation. For instance, the fusion leading to the formation of the Christian Democratic Appeal in the Netherlands in 1980 helped to eliminate the cleavage between Catholics and Protestants in the Dutch party system and substantially reduced party system fragmentation (Koole, 1994). Outside Europe, the merger between the Progressive-Conservative (PC) and the Canadian Alliance parties in 2003 created a new right-wing formation, significantly altering the Canadian party system and subsequent voting behavior (Bélanger and Godbout, 2010).

Despite the evidence showing that parties across the world are increasingly seen to join forces before elections (Powell Jr, 2000)—adopting various governance configurations—*pre-electoral* alliances have not received much attention from the literature. Most studies take the outcome of the electoral process as given and focus on government *post-electoral* coalitions. This paper follows a complementary approach: I consider the outcome of the post-electoral process as exogenous, and study a model of *electoral competition* in which parties can form pre-electoral coalitions and mergers before elections. The model predicts which features of the electoral environment facilitate the formation of each type of alliance. Understanding the incentives behind different pre-electoral alliances is crucial, as these have significant consequences on government composition, policies and party systems development.

The model features a two-period electoral game between three policy-motivated parties. Each party is associated with a fixed policy platform, which can be changed by joining a pre-electoral alliance. The platform resulting from an alliance is a convex combination of the constituent parties' platforms, with weights given by parties' electoral shares. In each period the centrist party decides whether to run alone or to propose an alliance to the leftist or rightist party: the proposal can be to merge or to form a pre-electoral coalition (hereafter, PEC).

A merger is a binding arrangement that solidifies the relative power (i.e., electoral weight) constituent parties have at a given point in time, and persists across elections. Conversely, PECs are only temporary alliances that need to be renegotiated in each period. That is, PECs preserve parties' identities and adjust platforms to parties' electoral weight in each period. This assumption reflects the empirical regularity that PECs are often revisited: indeed, parties composing coalitions typically change across elections.[4] In contrast, once a merger is formed there is a high cost for terminating it, and empirical evidence suggests that mergers persist more easily across elections, as shown in Table 1.

Number and Stability of Electoral Coalitions and Mergers per Country

Country	Average Number of Unstable Coalitions per Election	Average Number of Stable Coalitions per Election	Average Number of Unstable Mergers per Electoral Period	Average Number of Stable Mergers per Electoral Period
Bulgaria	1.5	0.7	0.2	0.3
Czech Republic	0.5	0.5	0.0	0.7
Estonia	1.0	0.8	0.3	1.3
Hungary	0.3	0.5	0.0	0.0
Latvia	0.7	1.5	0.2	1.0
Lithuania	1.0	0.2	0.3	1.2
Poland	1.8	0.5	0.2	0.8
Romania	1.0	0.3	0.0	1.2
Slovakia	0.7	1.0	0.0	1.0
Slovenia	0.2	0.0	0.2	0.7
Total	1.0	0.6	0.1	0.9

Table 1 – Stable/unstable coalitions and mergers in the first six electoral periods in 10 countries in Central and Eastern Europe. A stable electoral coalition is a coalition whose composition remains unchanged in two consecutive parliamentary elections. An unstable coalition is intact for one election only. Source: Ibenskas and Bolleyer (2018).

I first ask when parties compete *independently* at election time. In the baseline model, the policy implemented after the election is the one preferred by the party with a plurality of votes (i.e., if no party has a majority, a minority government forms). Since parties are policy-motivated, the centrist party never runs alone unless it has a plurality. However, having a plurality is not sufficient for parties to run alone in equilibrium. Parties are forward-looking, and between periods an exogenous shock to voters' preferences might change parties' relative vote shares. The centrist party is willing to run alone only when it has a plurality of votes in the first period *and* it is likely to maintain a plurality in the second period (i.e., when electoral

[4]There are notable exceptions, such as the stable coalition between the German Christian Democratic Union (CDU) and its sister party in Bavaria, the Christian Social Union (CSU).

volatility is low). Given that the centrist party is the pivotal actor, under these conditions no alliances form and parties run alone in equilibrium.

When the centrist party does not have a plurality, running alone is strictly dominated by proposing an alliance, which ensures to implement a policy closer to the centrist party's favorite one. In this case, which type of alliance do parties form in equilibrium? Parties face a *dynamic* trade-off: while mergers insure constituent parties against unfavorable shifts in the electorate's preferences, these binding forms of alliances come at the cost of losing the opportunity to join more advantageous coalitions in the future. Conversely, PECs offer more flexibility to respond to changes in voters' preferences.

Results show that the choice between mergers and PECs crucially depends on *electoral volatility* (i.e., the likelihood of large shifts in voters' preferences). When electoral volatility is high enough, in equilibrium parties form strong alliances such as mergers. Intuitively, if voters' preferences shift too much in one direction, the advantaged party can govern alone; hence for high realizations of the shock the centrist party risks being left out of power. Conversely, as voters' preferences become more stable, the centrist party values more flexibility and prefers to wait to form a more advantageous coalition in the future. Electoral instability is often considered a characteristic of the early years of democratic regimes (Kitschelt et al., 1999). This result provides an explanation for the empirical observation that the frequency of mergers decreases as democratic regimes mature (Ibenskas and Sikk, 2017).

When the electorate is sufficiently volatile, the centrist party might want the *more extreme* party as merger partner. Intuitively, the worst possible outcome for the centrist party is that the extremist becomes so popular to win an outright majority in the future. Since preferences are concave, the centrist outweighs this possibility and merges with the extremist, even when doing so means to forego the possibility to implement its bliss point in the current election. This result is robust to considering pre-electoral bargaining among constituent parties, where the merger platform is chosen by the centrist party. I assume that the centrist party makes a take-or-leave offer to join a merger to either the extreme or the moderate party. In equilibrium, the centrist party merges with the extremist for sufficiently high volatility.

How does the central trade-off of the model vary with different electoral, legislative, and executive institutions? An extension formalizes how the incentives to form pre-electoral alliances depend on inter-party power sharing (Lijphart, 1984). The degree of power sharing depends on both the rules mapping votes into seats (e.g., electoral rule proportionality) and the rules governing legislative decisions (e.g., the presence of super-majority requirements). The model predicts that some degree of power concentration is a necessary condition for both PECs and mergers to take place. This result is in line with the empirical literature showing that disproportional electoral systems can induce parties to join forces by forming pre-electoral alliances to maximize their electoral chances (Olsen, 2007; Rakner, Svåsand and Khembo, 2007; Bélanger and Godbout, 2010). Conversely, pre-electoral alliances are not sustainable in consensual democracies, where parties tend to compete more independently.

While PECs allow parties to campaign autonomously, mergers demand that parties give up their ideological identities by forming new political entities. If voters are uncertain about the exact location of parties' platforms, they might evaluate differently a merger and a PEC between the same parties. An extension of the model incorporates voters' uncertainty by introducing noise in the location of parties' platforms. To capture the fact that "mergers reduce, or even destroy, the information value of party labels for voters" (Ibenskas, 2016, 343), I assume that mergers are associated with higher noise than PECs, and the noise is increasing in the distance between the constituent parties' bliss points. The analysis shows that mergers are not sustainable in equilibrium for high values of ideological uncertainty.

This paper provides novel insights for the process of party system stabilization. The literature has often linked electoral volatility to unstable party systems.[5] However, by assuming that a volatile electorate is responsible for system instability, this approach overlooks the fundamental choices of elites, who strategically respond to electoral volatility when affecting the shape of the party system. In fact, the model shows that volatility does not necessarily lead to an unstable party system, and that it can increase stability with the creation of stable alliances.

[5]Indeed, several studies even use measures of electoral volatility as an indicator of party system instability (for an overview on Western European and OECD countries, see Tavits, 2008).

2. Contribution to the Literature

A small but growing empirical literature has begun to analyze different types of pre-electoral alliances (Golder, 2006; Ibenskas, 2016; Frey, López-Moctezuma and Montero, 2021; Hortala-Vallve, Meriläinen and Tukiainen, 2021). In her seminal contribution, Golder (2006) provides the first systematic study of PECs worldwide. Ibenskas and Bolleyer (2018) show that mergers in Central and Eastern Europe tend to be more stable than PECs. Both studies call for more systematic research on how mergers and PECs differ. Methodologically, I contribute to this literature by providing a new dynamic theory that differentiates between mergers and PECs. Substantially, I provide—to the best of my knowledge—the first empirical implication linking the role of electoral volatility to the incentives to form different pre-electoral alliances.

This paper belongs to the vast literature on electoral competition. While models of elections overwhelmingly focus on two-party competition, I provide a theory of the electoral process with multiple competing parties who can form alliances prior to elections. Close to this work is Shin (2019), who analyzes a static model of three candidates running for a single office in which only two candidates can form a coalition, and shows how factors such as electoral rules and ideological affinity affect the decision to form PECs. My model introduces both *dynamic* incentives and the possibility of distinct parties *merging* into a new political entity.

The model also relates to the literature on endogenous party formation. In particular, Levy (2004) analyses party formation in the presence of a multidimensional policy space, where policy-motivated *politicians* can form coalitions (parties) to credibly commit to a broader set of policies (the Pareto set of the coalition). I model *parties* forming coalitions, and focus on their dynamic trade-off. Rather than serving the role of *commitment* device, in my model stable coalitions (i.e., mergers) act as an *insurance* device against negative electoral outcomes. In Morelli (2004) as well, parties help voters to coordinate. He analyzes multi-district elections in a uni-dimensional policy space under different electoral rules. Snyder and Ting (2002) consider a fixed number of parties and analyze endogenous platforms, or brands, which allow candidates to signal their preferences to voters.

While this paper models the electoral process, it is also broadly related to the vast literature on post-electoral coalition formation (Baron and Ferejohn, 1989; Laver and Shepsle, 1990;

Austen-Smith and Banks, 1990). In a related model, Bandyopadhyay, Chatterjee and Sjostrom (2011) study how the *post-electoral* bargaining protocol—random recognition à la Baron and Ferejohn (1989) vis-à-vis a protocol rewarding the winner—affects parties' incentives to form pre-electoral coalitions. Rather than analyzing the equilibrium of the post-electoral bargaining process, I take post-electoral outcomes as exogenous and focus on how pre-electoral incentives are affected by the electoral environment.

3. The Model

Consider a two-period game of electoral competition between three policy-motivated parties: $i = \ell, c, r$. Each period features a proposal stage, which determines parties' alliances, and an election. Each party is associated with a preferred policy platform $z_i \in \mathbb{R}$, where $z_\ell < z_c < z_r$. There exists a continuum of voters, indexed by v, who vote for one of the parties. Voters' ideal points are uniformly distributed over a subset of the policy space, $\mathcal{Z} \equiv [-a, a]$, where $\mathcal{Z} \subset \mathbb{R}$.[6] The ideal policy of voter v is denoted by $z_v \in \mathcal{Z}$.

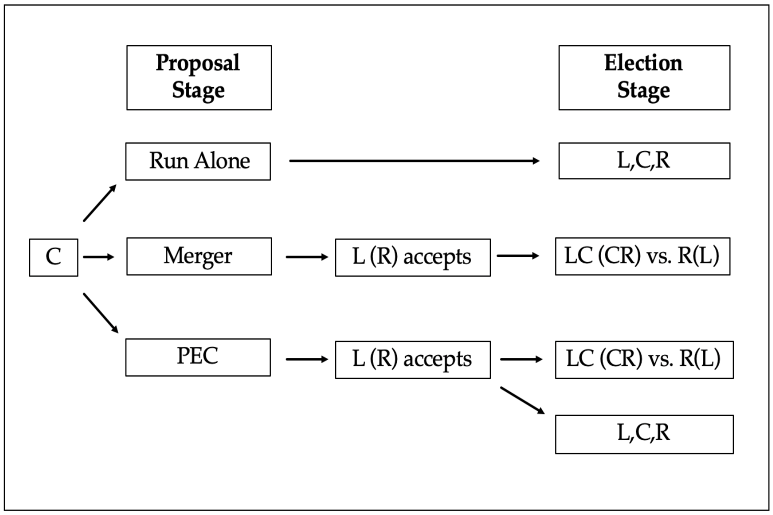
The sequence of the proposal stage is summarized in Figure 1. First, the centrist party c decides whether to run alone or not. If it runs alone, parties compete independently. If it does not run alone, c can propose to either ℓ or r to form a merger. If c's proposal to ℓ (r) is accepted, the merged party runs against r (ℓ). If c's proposal is rejected, or if no merger is proposed, c proposes a PEC to either party. If c's proposal to ℓ (r) is accepted, the PEC formed by ℓ, c (c, r) runs against r (ℓ). Otherwise, parties compete independently. After the proposal stage is completed, an election takes place, resulting in the adoption of a policy denoted $\hat{x}_t$.

The implemented policy $\hat{x}_t$ is the platform chosen by the winner of the election, i.e., the party, PEC or merger with the majority of votes in t. If no party/merger/PEC obtains a majority, the implemented policy is determined post-electorally by the party/merger/PEC with a plurality of votes, which forms a minority government.[7] This assumption is motivated by the frequency of minority governments both within and outside Europe (Strom, 1984).

[6] This assumption is without loss of generality and is merely convenient for computing parties' vote shares.

[7] Section 6 analyzes the case where the implemented policy is a compromise among the policy positions of all the parties composing the parliament, without regard to whether these parties are in government or opposition.

Figure 1 – Proposal Stage Sequence.



When party i wins the election running alone, the implemented policy is $\hat{x}_t = z_i$. When an alliance wins, the implemented policy is a convex combination of the constituent parties' bliss points. Suppose that ℓ and c merge (form a PEC). Then, the resulting policy platform is:

$$z^{m}_{\ell c} = z^{\text{pec}}_{\ell c} = \lambda\, z_\ell + (1-\lambda)\, z_c, \tag{1}$$

where the weight $\lambda \in (0,1)$ captures the relative strength of the extreme party. The policies resulting from an alliance between c and r are defined analogously. In the Appendix (Section 7), I analyze the case where weights depend on parties' vote shares (and as such change over time). While some of the results are more nuanced, the main trade-off that emerges from the analysis is analogous to this setup, where expressions are substantially simpler. Section 5 endogenizes the weight λ as the outcome of pre-electoral bargaining between the constituent parties. In this case as well the main results remain qualitatively unchanged.

Notice that the proposal stage rules out the possibility of an alliance between ℓ and r.[8] The sequence of the proposal is empirically motivated by the flexible nature of PECs vis-à-vis

[8]Besides being empirically rare, it is not clear which platform would emerge from an alliance between two non-moderate parties at the opposite extremes of the ideological spectrum, nor how to compute the resulting vote share.

mergers: c can propose a PEC to either ℓ or r *after* a merger proposal has been rejected, while it cannot propose a merger to ℓ (r) after a merger proposal to r (ℓ).[9]

Denote by $V_{i,t}$ party i's vote share at time $t = 1, 2$, where $V_{i,1} < 1/2$ for each party i. At the beginning of the second period ($t = 2$), an exogenous shock ξ favoring party r affects all voters equally, where ξ is uniformly distributed in $\left[-\frac{1}{\psi}, \frac{1}{\psi}\right]$. A positive (negative) realization of the shock shifts voters' ideal points to the right (left). The support of the shock represents electoral volatility: as ψ decreases, the support of the shock becomes larger, and electoral volatility increases. Conversely, as ψ increases, the support of the shock shrinks and the electoral outcome becomes more predictable.

After the shock is realized, if no merger formed in $t = 1$ the proposal and election stages of the second period take place. To simplify the description of the equilibrium, I assume that mergers persist in $t = 2$ after being formed in $t = 1$. That is, constituent parties cannot split in the period that follows the merger formation. This assumption is motivated by the bureaucratic and electoral costs that mergers might cause. Typically, several legal requirements are needed for the registration of a new party, which could impede the formation of a splinter party following a recent merger (Hug, 2001). Voters' preferences might also change: Previous supporters of the constituent parties might transfer their loyalties to the merged party. Furthermore, voters might consider the members of the splinter party as noncredible because of frequent changes in their party affiliation (Mershon and Shvetsova, 2013).

Voters and parties have quadratic preferences over policies. Voter v's realized payoff from the implemented policy $\hat{x}_t$ is defined as $u_v(\hat{x}_t) = -\left(z_v - \hat{x}_t\right)^2$. Similarly, party i's payoff from $\hat{x}_t$ is $u_i(\hat{x}_t) = -\left(z_i - \hat{x}_t\right)^2$. I focus on subgame perfect equilibria in pure strategies. A pure strategy for party c defines c's decision to run alone, form a merger with ℓ (r), or form a PEC with ℓ (r) in $t = 1$, as well as in $t = 2$ conditional on no mergers forming in $t = 1$. For party ℓ (r) a pure strategy defines (i) an acceptance decision following a merger proposal in $t = 1$ and, conditional on no merger proposals being made, an acceptance decision following a PEC proposal to ℓ (r); (ii) if no mergers formed in $t = 1$, an acceptance decision following

[9] An alternative (less credible) bargaining protocol would allow c to make sequential merger proposals. However, this would not qualitatively affect the main results. Similarly, having c proposing a PEC to only one party or to both does not qualitatively change the results.

a merger proposal to ℓ (r) in $t = 2$ and, conditional on no merger proposals being made, an acceptance decision following a PEC proposal to ℓ (r). Voters are myopic, and since no voter is ever pivotal, I adopt the standard assumption that voters vote sincerely. Parties, on the other hand, maximize their expected overall payoff, and each party evaluates the future according to a common discount factor $\delta \in (0,1)$.

4. Results

To begin, consider the incentives to form alliances in the second period, when no merger formed in the first period. To simplify the notation, assume that $z_c = 0$, $z_r = 1$ and $z_\ell \in (-1, 0)$.

Absent dynamic considerations, party c simply compares the payoffs from running alone and from forming an alliance. Recall that by assumption the realized policy is set by the party (or coalition) with the plurality of votes. When the centrist party has a plurality, it can set its preferred policy by running alone. In this case no alliances form and $\hat{x}_2 = z_c$. When c does not have a plurality, on the other hand, running alone is dominated and c proposes an alliance. Because $z^m_{\ell c} = z^{\text{pec}}_{\ell c}$ ($z^m_{cr} = z^{\text{pec}}_{cr}$), c is indifferent between merging and forming a PEC. I assume that, when indifferent, parties choose PECs over mergers.

When can the centrist party successfully form a PEC in the second period? First, for a coalition to be incentive compatible *neither party should have a majority of votes*, otherwise the majority party could implement its preferred policy both by running alone and by facing an opposing PEC. Furthermore:

Remark 1. *A necessary condition for the emergence of PECs in equilibrium is that $V^{pec}_{\ell c,2} > 1/2$ and $V^{pec}_{cr,2} > 1/2$.*

That is, both PECs need to obtain a majority if formed. Suppose this is not the case, i.e., $V^{\text{pec}}_{\ell c,2} > 1/2 > V^{\text{pec}}_{cr,2}$ which implies $V_{\ell,2} > \max\{V_{c,2}, V_{r,2}\}$.[10] If c were to propose a PEC to ℓ, ℓ would reject because it could set its preferred platform by forming a minority government after the election.[11] Similarly, a PEC between c and r would not change the post-electoral

[10] That is, ℓ could have a majority running against a PEC but not if all parties were to run alone. This is because, for any $\lambda > 0$, when a PEC between c and r forms some centrist voters turn to ℓ.

[11] Notice that this would not be true if the realized policy was determined after parliamentary negotiations among all parties. Section 6 analyzes this possibility.

policy set by ℓ. Hence, when only a PEC between ℓ and c reaches the absolute majority of votes, in equilibrium parties run alone and ℓ forms a minority government (the case such that $V_{cr,2}^{\text{pec}} > 1/2$ is analogous).

When both PECs obtain a majority of votes, forming a PEC is incentive compatible for both ℓ and r. Rejecting an alliance is strictly dominated for both parties because of the threat of c forming a PEC with the opposite party. The following proposition shows when parties form alliances or run alone as a function of the shock realization, which determines the vote shares in the second period. The values of the shock realization are derived in the Appendix and define for which vote shares PECs are incentive compatible for all parties: when both PECs obtain a majority (i.e., for intermediate realizations of the shock), c proposes a PEC to the closest party ℓ because of its closer platform.

Proposition 1. Second-Period Policy. *If $V_{c,2} > V_{\ell,2}, V_{r,2}$, in equilibrium parties run alone in the second period. Suppose that c has no plurality, and that no merger formed in the first period. Then:*

- *for $\xi < \frac{z_\ell+\lambda}{2}$, parties run alone and $\hat{x}_2 = z_\ell$,*
- *for $\frac{z_\ell+\lambda}{2} < \xi < \frac{1+\lambda z_\ell}{2}$, a PEC between c and ℓ forms,and $\hat{x}_2 = z_{\ell c}^{pec}$,*
- *for $\xi > \frac{1+\lambda z_\ell}{2}$, parties run alone and $\hat{x}_2 = z_r$.*

Proof. All proofs can be found in the Appendix. □

One interesting insight that emerges from Proposition 1 is that parties can join PECs to prevent other parties from winning the election and obtaining control of the policy-making process. In line with this logic, Hortala-Vallve, Meriläinen and Tukiainen (2021) provide evidence from Finnish municipalities that parties join PECs to avoid concentration of power in the hands of the largest party when this is close to obtaining more than half of the seats. Similarly, Frey, López-Moctezuma and Montero (2021) document that in Mexican mayoral elections parties form alliances to remove advantaged incumbent parties from office.

We now turn to the first period. Similarly to the second period analysis, let us first consider whether c has a plurality of votes in the first period. If this is not the case, then running alone is clearly dominated, because by running alone c would suffer a higher policy cost (from the platform of the party with a plurality) than from forming an alliance. Hence in this case c never runs alone in equilibrium. If $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$, on the other hand, c faces a trade-off. Running alone is preferred in the first period because it would allow c to set its preferred policy. So, if parties were myopic, in equilibrium c would run alone when having a plurality of votes. Yet, parties consider their expected second period payoff, which is affected by electoral volatility. The following analysis considers this trade-off and shows under what conditions parties run alone in equilibrium.

When do Parties Run Alone?

Suppose that c has a plurality of votes in the first period. In this case, proposing a PEC is clearly dominated: the second period expected payoff is identical to that of running alone because PECs are only temporary alliances, and $u_c(z_c) > u_c(z^{\text{pec}}_{\ell c,1}), u_c(z^{\text{pec}}_{cr,1})$ because c has a plurality and could implement its preferred policy. Therefore, c compares the expected payoff from running alone to that of forming a merger.

Between the two elections, voters' preferences change. The magnitude of this change is crucial to determine whether c prefer to run alone or merge with a different party. If volatility is low, c expects to keep its majority status. While both ℓ and r prefer to join the centrist to set a more favorable policy, c prefers to run alone, given the high chances of a future solo victory.

When volatility is high, on the other hand, the fear of losing popularity in the future might lead c to form a merger, as an insurance against an unfavorable change to voters' preferences. Recall that $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. This assumption implies that we are stacking the deck against the emergence of mergers in equilibrium: mergers are more costly when c has a plurality of votes because of the need to forego a first-period solo-rule. The next result shows that when the electorate's future choices are easily predictable, party c prefers to run alone and in equilibrium does not propose any alliance, whereas when volatility is high c prefers to merge, thus losing the opportunity to implement its preferred policy in the first period.

Suppose that volatility is sufficiently high so that c wants to merge in equilibrium. Which party does c prefer to merge with? The reader might expect that c always prefer a merger with the closer party ℓ. Indeed, the payoff that c obtains in the first period from merging with ℓ is higher than the one following a merger with r, because the implemented policy resulting from the former is closer to c's ideal point. However, this is not always the case: depending on how volatile the electorate is, c might prefer to merge with r. Proposition 2 shows that as volatility increases, c prefers to merge with the ideologically more distant party (r).

Proposition 2. First Period - Equilibrium. *Let $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. There exists $\hat{\psi}^a$ such that c is indifferent between merging and running alone. Furthermore, there exists $\tilde{\psi}$ such that c is indifferent between merging with ℓ or r. In equilibrium:*

- *for $\psi > \hat{\psi}^a$, parties run alone and $\hat{x}_1 = z_c$,*
- *for $\tilde{\psi} < \psi < \hat{\psi}^a$, a merger between c and ℓ forms, and $\hat{x}_1 = z^m_{\ell c}$,*
- *for $\psi < \tilde{\psi}$, a merger between c and r forms, and $\hat{x}_1 = z^m_{cr}$.*

A merger represents an insurance against an unfavorable outcome in the second period. The worst expected outcome for c is that the electorate moves to the right so that r obtains a majority of votes and $\hat{x}_2 = 1$, r's preferred platform. Since r is ex-ante disadvantaged, this event can only occur under high electoral volatility. If c were to merge with the closest party ℓ, the first period payoff would be higher than the payoff from merging with r. However, c would run the risk of losing against r in the second period, even if running as a merged party. Since preferences are concave, the latter policy cost following a victory of r carries more weight. As volatility increases, the worst case scenario becomes more likely, and c prefers to insure itself by merging with the more distant party r, thus giving up a first period solo rule and losing some moderate voters to ℓ.

The finding that the centrist party might form a merger with the more distant r to prevent its victory in the future might seem counter-intuitive. This result follows from the role of concavity and from the assumption that the platform implemented by the merged party—a convex combination of c and ℓ's bliss points —pulls some centrist voters towards the extreme party r. The result is also robust to an extension (see Section 5) where c makes a take-or-leave offer $\lambda \in [0, 1]$ to ℓ (r), thus showing how the qualitative result in Proposition 2 is unchanged when allowing for pre-electoral bargaining.

An interesting question is how the trade-off between the two merger partners changes as their relative ideological extremism varies.

Remark 2. Ideological Extremism. *Let* $\Delta_c^m(\psi) = U^m_{c,\ell c} - U^m_{c,cr}$ *define party* c*'s net expected payoff from merging with* ℓ*.*

- $\Delta_c^m(\psi)$ *is increasing in* z_ℓ *for* $\lambda > 1/2$*,*
- $\Delta_c^m(\psi)$ *is decreasing in* z_ℓ *for* $\lambda \leq 1/2$*,*
- $\partial^2 \Delta_c^m(\psi)/\partial\psi\partial z_\ell > 0.$

As ℓ moves closer to c's preferred platform, c is more likely to merge with the closer ℓ than r. This, however, only holds when c has *less* weight in the final platform of the merged party ($\lambda > 1/2$). When c has more weight ($\lambda \leq 1/2$), the worst second-period outcome conditional on a merger between ℓ and c—i.e., a solo victory by party r—remains the same as z_ℓ becomes more moderate, while the prospect of a merger victory gets only slightly better. Conversely, the worst second-period outcome conditional on a merger between c and r—i.e., a solo victory by party ℓ—becomes less costly for c as z_ℓ becomes more moderate. Because of concave preferences the latter has more weight than the former and, all else equal, c prefers to merge with r as ℓ becomes more moderate.

Finally, from Proposition 2 we know that c's incentives to merge with the extreme party r increase with electoral volatility. Remark 2 shows that the magnitude of this incentive varies with the relative extremism of r: as z_ℓ moves towards zero, r becomes relatively more extreme. The positive cross-partial thus implies that an increase in volatility *increases* the region of the parameter space supporting an equilibrium where c merges with the extreme party more when r is relatively *more extreme*. Intuitively, when the extreme party is more of a threat for c, because it could win an absolute majority by itself, then c is more prone to join it to prevent its solo victory in the future. Conversely, as z_ℓ moves away from the center, r becomes relatively more moderate and the advantage of merging with r vis-à-vis ℓ shrinks. Figure 2 provides an illustration of these results, plotting the region such that $\Delta_c^m(\psi) > 0$ as a function of ψ (x axis) and z_ℓ (y axis), for $\lambda = 0.4$.

The Trade-off between Mergers and PECs

Suppose that c does not have a plurality of votes in the first period. What conditions can sustain an equilibrium in which parties merge? First, notice that c would never run alone or

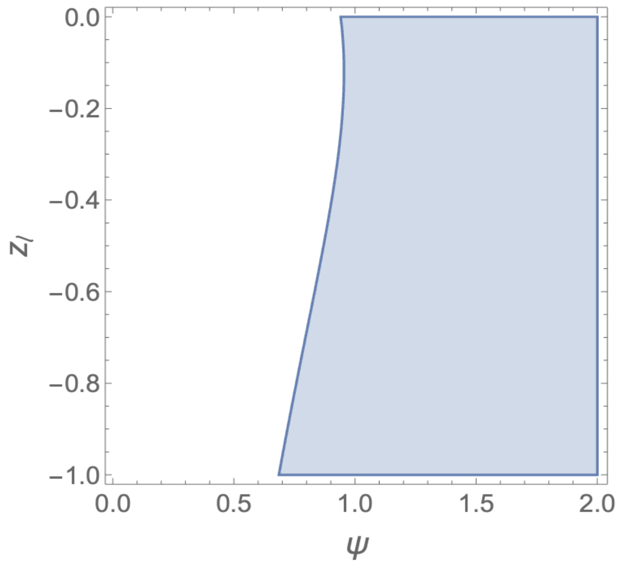


Figure 2 – Merger decision. $\Delta_c^{\mathrm{m}}(\psi)$ as a function of the value of ψ and z_ℓ. The blue region corresponds to the values of ψ, z_ℓ such that c prefers to merge with ℓ rather than with r ($\Delta_c^{\mathrm{m}} > 0$). For low values of ψ (i.e., high volatility), c prefers to merge with r (bottom left region), even if the latter is further away from c. As ℓ gets closer to c, c prefers a merger with r, ceteris paribus. The other parameters are set to $a = 1$, $\delta = 0.8$ and $\lambda = 0.4$.

form a PEC with r in equilibrium, because both options are strictly dominated by forming a PEC with ℓ. Thus, c compares the expected payoffs from the two mergers to that of forming a PEC with ℓ.

The choice between mergers and PECs depends on electoral volatility. On the one hand, the centrist party wants to insure itself against negative electoral shocks. Mergers provide such an insurance, by tying the centrist policy to a common platform which has higher chances to be implemented in the future. On the other hand, c values flexibility, that is the possibility to form a future coalition with the best possible partner, given the future electoral conditions.

Lemma 1. *There exists $\hat{\psi}$ such that c is indifferent between merging and forming a PEC with the closest party (ℓ). Then:*

- *for $\psi < \hat{\psi}$, c prefers to merge with ℓ,*
- *for $\psi > \hat{\psi}$, c prefers to form a PEC with ℓ.*

Lemma 1 shows that high values of volatility call for mergers, while as the electorate becomes more stable the centrist party values more the flexibility provided by PECs. We can now describe the equilibrium of the baseline game when c does not have a plurality of votes. The next result shows that different alliance configurations emerge in equilibrium, depending on electoral volatility. Recall that $\tilde{\psi}$ defines the value of ψ such that c is indifferent between merging with ℓ or r. We have:

Proposition 3. Electoral Volatility and Pre-Electoral Alliances. *Let* $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$. *In equilibrium:*

- *for* $\psi < \tilde{\psi}$, *a merger between* c *and* r *forms, and* $\hat{x}_1 = z^m_{cr}$,
- *for* $\psi > \tilde{\psi}$, *a PEC between* ℓ *and* c *forms, and* $\hat{x}_1 = z^{pec}_{\ell c}$.

Proposition 3 provides a rationale for parties' incentives to join different types of alliances. When the likelihood of large shifts in voters' preferences is high enough, in equilibrium the centrist party prefers to merge rather than to form a PEC. By merging, the centrist party insures itself against large shifts in the electorate's preferences at the cost of losing the opportunity to form a more advantageous coalition in the future. When the electorate is highly unpredictable ($\psi < \tilde{\psi}$), risk-aversion considerations trump the first period policy cost, resulting in a merger with more distant allies. As Proposition 2 shows, this happens because c wants to insure itself against the worst possible outcome (i.e., r implementing its bliss point).

Mergers are not sustainable anymore when voters' preferences are stable—which can be empirically associated with a highly partisan electorate. In this case, the centrist party values more flexibility, and forms with the closest party a temporary alliance which does not bind its policy platform in the future. By forming a PEC in the first period, the centrist party maintains its identity, preserving its brand for the future election, when more information about voters' preferences is available.

Finally, note that the centrist party never merges with ℓ in equilibrium, even though for some values of volatility ($\tilde{\psi} < \psi < \hat{\psi}$) c would prefer to do so. This follows from the merger protocol assumption: i.e., c cannot propose to merge after a previous merger proposal has been rejected. Assuming that c can sequentially propose a merger to both parties does not change the parameter space for which parties form mergers in equilibrium. However, in this case c would merge with ℓ for $\tilde{\psi} < \psi < \hat{\psi}$, and with r for $\psi < \tilde{\psi}$.

Figure 3 provides a graphical representation of the equilibrium, illustrating which types of alliances (if any) emerge in equilibrium as a function of electoral volatility: the top (bottom) panel assumes that $V_{c,1} < (>) \max\{V_{\ell,1}, V_{r,1}\}$. In the top panel, the dark gray region plots

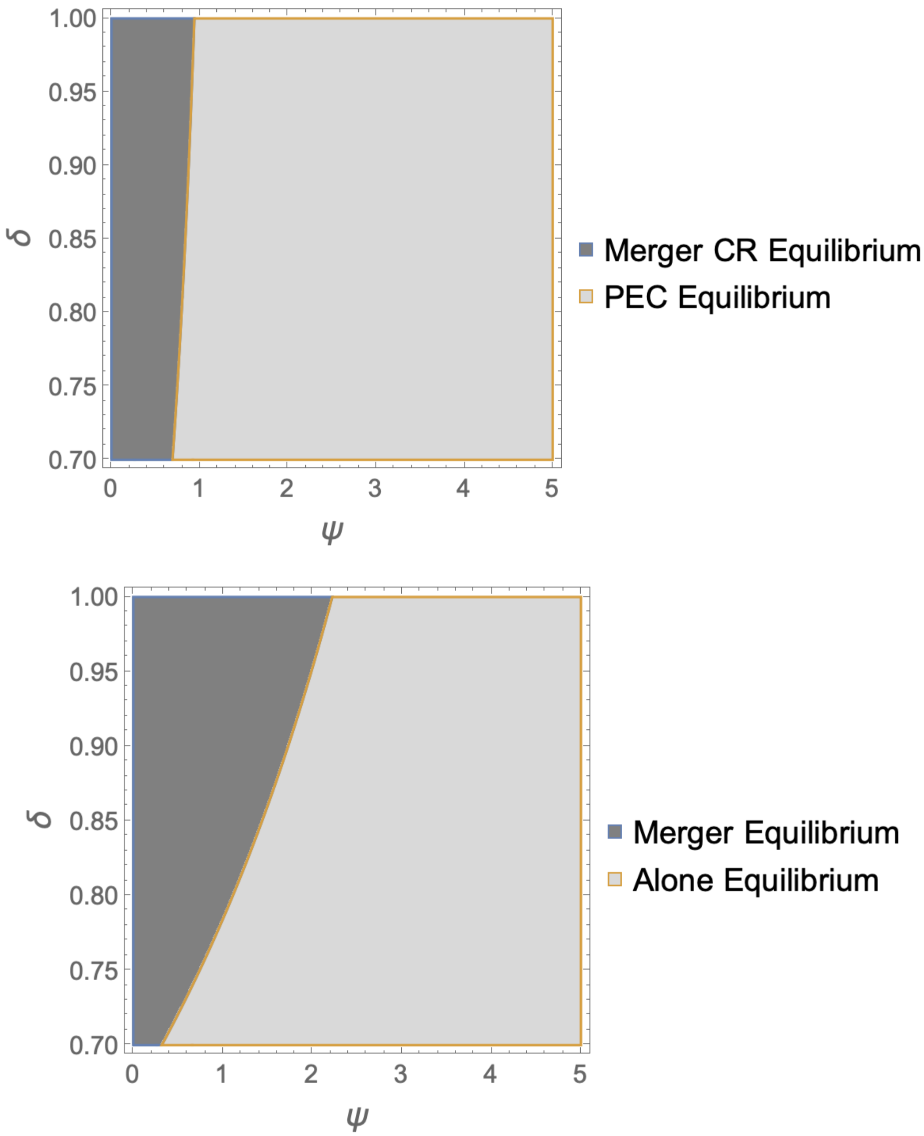


Figure 3 – Top Panel: Equilibrium featuring a merger between c and r (dark gray region) and a PEC between ℓ and c (light gray region) as a function of ψ (x axis) and parties' discount factor δ (y axis), for $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$. Bottom Panel: Equilibrium featuring merger (dark gray region) and parties running alone (light gray region) for the same parameter space. Parameters are set to $z_\ell = -0.7$, $a = 1.5$, $\lambda = 0.5$, for $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$.

the range of parameters sustaining an equilibrium where c and r merge in the first period, while the light gray region plots the range for which ℓ and c form PECs in the first period, as a function of ψ (x axis) and parties' discount factor (y axis). Being electorally advantaged, ℓ always prefers a PEC to a merger with c. Hence, when volatility is low (ψ high enough), c proposes a PEC to ℓ, which accepts, and a PEC forms in equilibrium (light gray region). As electoral volatility increases (ψ decreases), the centrist party's incentives to merge increase, and a merger forms for $\psi < \tilde{\psi}$. Similarly, the bottom panel shows that mergers form in equilibrium for high volatility (dark gray region) even when c has a plurality of votes and could set its preferred policy by running alone. Conversely, when volatility is low c prefers to run alone. Finally, note that the boundary between the two regions is not exactly vertical. A lower discount factor mutes the extent to which more electoral volatility results in a merger equilibrium. A binding alliance requires patience, because c could set its preferred policy in $t = 1$ but suffer a high policy cost in $t = 2$.

5. Pre-Electoral Bargaining

The baseline model assumes that coalition platforms are weighted averages of constituent parties' ideal points. This assumption abstracts from a bargaining process over the platform content, which likely takes place among constituent parties before they present their common platform. Considering such process is relevant to prove the robustness of the main mechanism of the paper, which shows that when the electorate is highly unpredictable, in equilibrium the centrist party forms a merger as an insurance device against negative shock realizations, and can even choose the more extreme party as a coalition partner.

One might think that the latter result fails to be robust to pre-electoral bargaining. That is, the centrist party could choose a sufficiently moderate platform that makes a future r victory unlikely, and such that party ℓ is indifferent between merging with c and running alone. In fact, I show that this is not the case: in equilibrium, the centrist party merges with the more extreme r even when coalition platforms are endogenous. I now formally introduce a simple bargaining protocol to lay out the intuition behind the result.

Suppose that ℓ and c merge (form a PEC). Following Levy (2004), I assume that the resulting policy platform belongs to the Pareto set of the coalition formed by ℓ and c, i.e.:

$z_{\ell c}^{m} = z_{\ell c}^{\text{pec}} = \lambda z_\ell + (1-\lambda) z_c$, where the weight $\lambda \in [0,1]$ is *endogenous* and chosen by party c. In particular, c makes a take-or-leave offer to ℓ, which can either accept or reject. Acceptance leads to the formation of a merger. Rejection leads to the following step of the proposal stage, which remains unchanged.

Let us focus on the case where c already has a plurality of votes in the first period. Under this assumption, the formation of mergers in equilibrium is harder to sustain because of c's myopic incentives to run alone, thus it represents a harder test. The following result shows that for high volatility c does merge with the extreme party, and that in equilibrium there are policy concessions ($\lambda^* > 0$).

Proposition 4. Bargaining. *Let $V_c, 1 > \max\{V_{\ell,1}, V_{r,1}\}$, and suppose ℓ is sufficiently moderate. There exists $\hat{\hat{\psi}}$ such that c is indifferent between running alone and merging with ℓ. In equilibrium:*

- *for $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, a merger between c and ℓ forms, and $\lambda^* = 0$,*
- *for $\psi < \tilde{\psi}$, a merger between c and r forms, and $\lambda^*(\psi, z_\ell) \in (0,1)$,*
- *for $\psi > \hat{\hat{\psi}}$, parties run alone and $\hat{x}_1 = z_c$.*

As in the baseline model, party c prefers to merge rather than running alone for high values of volatility, because mergers represent an insurance against negative shock realizations. Suppose that, when $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, c proposes a merger to ℓ with platform $\lambda z_\ell = 0$. Figure 4 illustrates the values of ψ, λ such that c and ℓ prefer to merge than running alone. As the left panel of Figure 4 illustrates, c (ℓ) prefers a merger to running alone for high (low) volatility, and there exists a parameter configuration such that a merger with platform equal to z_c forms. Are other values of λ sustainable? As λ increases, the merger platform shifts to the left, thus losing moderate votes to r. Furthermore, as the platform weight of ℓ increases, all else equal c prefers to run alone, thus c's incentive compatibility condition becomes more binding, as shown in the right panel of Figure 4.

Suppose $\psi < \tilde{\psi}$ (such that, from Proposition 2, c prefers a merger with r). For this range of volatility c needs to offer a platform λ^* that makes r indifferent between merging and running alone. When λ is low, a merger is not incentive compatible for r, which all else equal

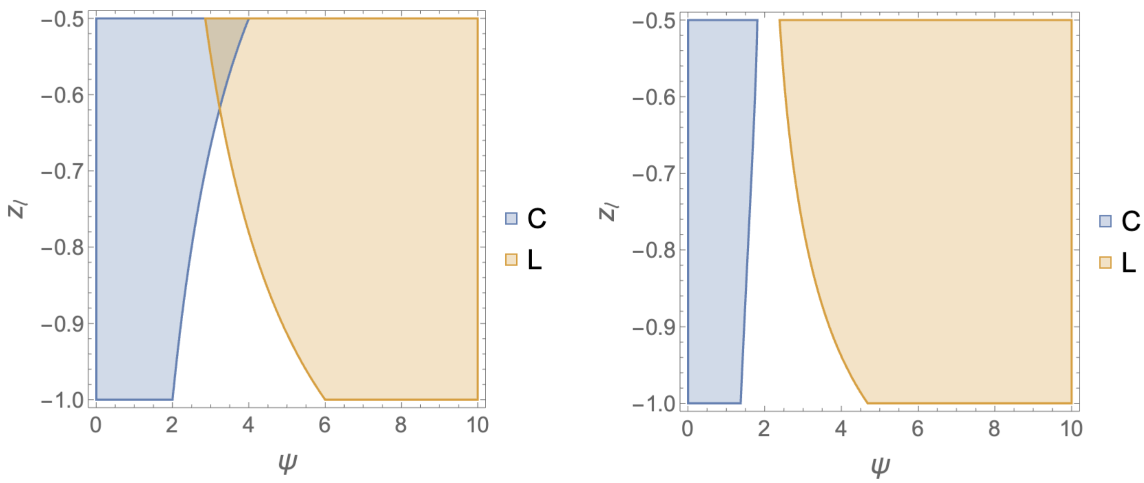


Figure 4 – Values of ψ (x axis) and z_ℓ (y axis) sustaining a merger between ℓ and c in equilibrium for $\delta = 0.7$. In the blue region, a merger is incentive compatible for c. In the orange region, a merger is incentive compatible for ℓ. In the left panel, $\lambda = 0$. In the right panel, $\lambda = 0.2$.

prefers a higher weight of its preferred policy in the merger platform. As λ increases, r's incentive compatibility constraint relaxes and r is willing to merge for more values of z_ℓ and ψ. Intuitively, a higher weight in the merged platform trumps the advantages of running alone. As λ increases, however, party c is worse off and less likely to merge with r. Figure 5 shows that there exist parameter values such that an equilibrium where the centrist party merges with the extreme party r can be sustained. In the left panel, $\lambda^* = 0.25$ and a merger is not incentive compatible for r. In the right panel, where $\lambda^* = 0.45$, r's incentive compatibility constraint relaxes and r is willing to merge for more values of z_ℓ. Otherwise, r is better off running alone.

The Threat of Entry. Suppose that, following a merger between c and r in $t = 1$, there are enough right extreme voters who would prefer a new (right) extreme party to the merged party. Alternatively, suppose that the extreme faction of the right party splits from it after the merger in $t = 1$, and competes alone in $t = 2$. We can introduce this threat of potential entry in reduced form, as a constraint that parties take into account when deciding whether to merge and with which platform.

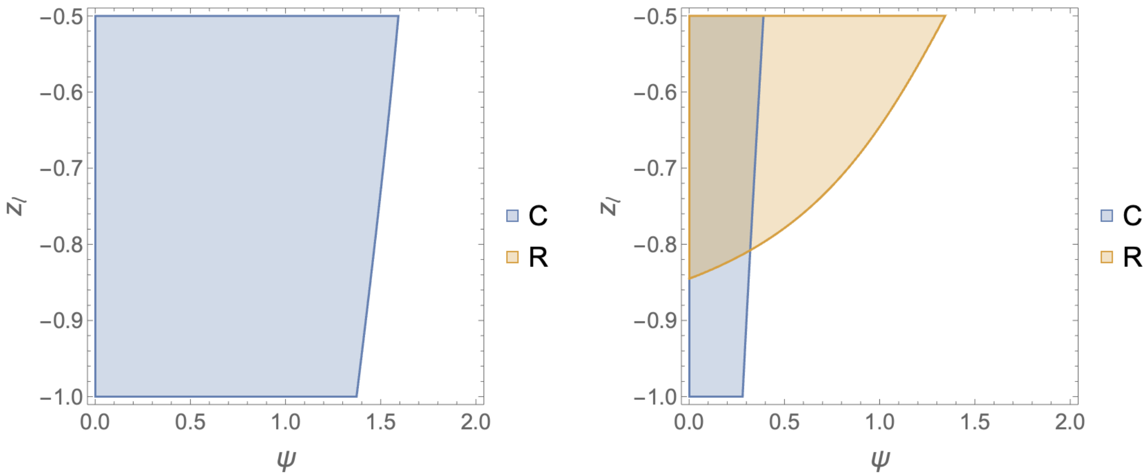


Figure 5 – Values of ψ (x axis) and z_ℓ (y axis) sustaining a merger between c and r in equilibrium for $\delta = 0.7$. In the blue region, a merger is incentive compatible for c. In the orange region, a merger is incentive compatible for r. In the left panel, $\lambda = 0.25$. In the right panel, $\lambda = 0.45$.

Suppose that, whenever c proposes to r a merger platform with $\lambda < 1$ and r accepts, the right extreme faction splits and forms a new party (or a new extreme party enters the race).[12] Denote by z_E the new "Extreme party" platform, where $z_E = z^m_{cr}(\lambda) + \kappa$, with $\kappa > 0$. Then, parties' vote shares in $t = 2$ are a function of the shock realization and κ: as $\kappa \to 0$, $z_E \to z^m_{cr}$. Empirically, a low value of κ corresponds to a strong extreme faction which is likely to have followers after splitting. Analogously, a low κ reflects a low cost of forming a new party.

Since λ is endogenously chosen by c, it is easy to see how the centrist party internalizes the threat of entry in its trade-off: One the one hand, a low λ is myopically preferred. On the other hand, proposing a high λ reduces the threat of the extreme faction splitting. It is straightforward to show that the equilibrium described in Proposition 4 is robust to the threat of entry for certain values of κ: when κ is sufficiently high (unless the extreme faction poses a significant threat) parties merge in equilibrium.

[12]If $\lambda = 1$, then the merger adopts an extreme platform that makes the extreme faction willing to stay within the new party.

6. Alternative Power Sharing Arrangements

The baseline model assumes that the party with the plurality of votes entirely controls the policy-making process. Empirically, this assumption corresponds to plurality voting systems (e.g., first past the post) and to the occurrence of minority governments. However, implemented policies could also result from post-electoral compromise among the policy positions of multiple parties composing the legislature. In consensual democracies, multiple parties typically exercise or have the potential to exercise significant policy influence (Lijphart, 1984).

This section varies the extent to which government policies reflect power-sharing among all parties as opposed to being determined by a single party. The degree of power sharing depends on both the rules mapping votes into seats (e.g., electoral rule proportionality) and the rules governing legislative decisions (e.g., which party is selected to be the *formateur*, or the presence of super-majority requirements). A decrease in power sharing might refer to a change in the electoral system (e.g., from proportional to winner-take-all), or to an institutional change holding fixed the electoral system's proportionality (e.g., from legislative-executive balance to executive dominance).

Alternatively to the baseline model—and at the other extreme—I now analyze the case where the implemented policy is a compromise among the policy positions of all the parties composing the parliament, without regard to whether these parties are in government or opposition, weighted by their seat shares. Proposition 5 below demonstrates that under this assumption no type of pre-electoral alliance is sustainable (neither PECs nor mergers) and in equilibrium parties always run alone. Thus, the model predicts that some degree of power concentration is a necessary condition for both PECs and mergers to take place.

Let the implemented policy be a function of parties' platforms (z_i) and their legislative power, measured by seat shares. For simplicity, I assume that parties' seat shares are exactly proportional to vote shares, or in other words that the electoral system is perfectly proportional.[13] Then, the implemented policy function is

$$\hat{x}_t(z_i) = \sum_{i=l,c,r} V_{i,t} \times z_i. \tag{2}$$

This formulation reflects the weight each party has in the post-electoral bargaining process in the legislature. The next result describes the equilibrium of the game under the parliamentary-mean assumption over policy outcomes.

Proposition 5. Parliamentary-Mean Equilibrium. *Let the implemented policy be an average of all parties' preferred policies, weighted by parties' vote shares. In equilibrium, neither mergers nor PECs are sustainable, and parties run alone in both periods.*

Proposition 5 shows that institutions that promote compromise and power-sharing among political parties reduce parties' incentives to join pre-electoral alliances to have their platform counted in the implemented policy. Under consensual political institutions there is no legislative premium for the winner of the election, hence parties compete alone.

To see why no pre-electoral alliance emerges in equilibrium, let us analyze first parties' static decision to form PECs vis-à-vis running alone. Contrary to the baseline model, where the implemented policy is determined by the winner of the election, the implemented policy under the parliamentary mean model reflects parties' compromise taking place after the election. Statically, the centrist party has no incentive to join a coalition given its advantaged position: because of concavity, c is always better off running alone. In the second period, conditional on an extreme shock realization c might prefer a PEC with the advantaged party. However, this is not incentive compatible for the advantaged party, who is always better off running alone. Thus, static incentives always induce parties to run alone.

Can dynamic incentives lead to the emergence of mergers? The proof of Proposition 5 shows that there exist a parameter configuration such that c prefers to merge rather than run-

[13] The degree to which a PR system resembles perfect proportionality in reality depends on many factors such as district magnitude (i.e. the number of seats awarded per district) and the existence (or absence) of electoral thresholds defined in terms of a minimum percentage of the national vote a party must win in order to guarantee parliamentary representation (Shugart and Taagepera, 1989; Cox, 1997; Lijphart, 2012). Among the most perfectly proportional systems are those of Israel, the Netherlands, and the Scandinavian countries (Lijphart, 2012).

ning alone when electoral volatility is high. This result is due to the concavity of parties' preferences over policies: by merging, the centrist party could prevent a higher policy cost due to one of the extreme parties' policies being weighted more. However, mergers are not incentive compatible for neither ℓ or r, which always prefer to run alone for any value of electoral volatility. Thus, in equilibrium no merger forms in the first period and parties compete alone in both periods.

How do different institutional arrangements influence policies? In "Elections as instruments of democracy: Majoritarian and proportional visions," Powell argues that power sharing institutions lead to *more moderate* policy outcomes then centralized party governments—because in consensual democracies opposing parties affect the policy making process. The results in this paper help to qualify this statement, showing how policy outcomes under different institutions vary depending on the equilibrium configurations of pre-electoral alliances. That is, in line with Powell Jr (2000), in a consensual democracy where an extreme party has a plurality of votes, the implemented policy is more moderate than the counterfactual policy in a centralized system *when no alliances emerge in equilibrium*. However, under a majoritarian system, mergers could also lead to centrist and stable policies, when the electoral conditions facilitate the formation of mergers (i.e., when electoral volatility is high).

7. Conclusion

The majority of multi-party systems are extremely "liquid" (Powell Jr, 2000; Golder, 2006): parties split, merge, form and leave coalitions at all times, and these movements largely affect parties' electoral chances. While the literature typically assumes that each party is associated to a particular policy platform—highlighting the important role of parties in producing political brand names (Downs, 1957; Snyder and Ting, 2002)—, in multi-party systems each party is often associated to different brands depending on the allies chosen. To understand the incentives behind different types of coalitions, this paper presents a model of electoral competition in which parties form alliances *before* elections, and decide *how binding* these alliances should be.

The central result of the model is that parties merge when voters are expected to be very volatile, because the centrist party want to insure itself against costly electoral outcomes. Importantly, this result does not depend on whether the centrist party is the major one or not: parties merge both when c has the plurality of votes (Proposition 2) and when it is a minority party (Proposition 3). This result is robust to considering the possibility of (pre-electoral) bargaining over the merger platform (Proposition 4), and to assuming that mergers are costly (Proposition C-1). Recent political developments have brought attention to the electoral decline of established parties and the burst of electoral volatility following the Great Recession of 2007 in Europe. The model suggests that this increased electoral volatility might lead to an increase in the number of binding coalitions in the future.

Results also show that at least some degree of power concentration is needed to trigger mergers and pre-electoral coalitions. Under consensual democracies that share power among all parties, minority parties do not need to join pre-electoral alliances to have their voices heard in the policy-making process. As power gets more concentrated in the hands of the winner of the election, parties need to join forces and both PECs and mergers can emerge in equilibrium. Consensual democracies are typically associated with more moderate policy outcomes. The model suggests that this can also be the outcome under majoritarian institutions when the electoral conditions call for mergers.

Results suggest that we should expect mergers to be empirically associated with volatile electorates, because volatility incentivizes parties to form strong alliances. In principle, an accurate measure of electoral volatility should reflect the extent to which personal votes change between subsequent elections. Thus, individual level data identifying voters' intentions to vote or party identification across time represents an accurate measure of volatility.

This paper begins to unpack the incentives behind different forms of pre-electoral alliances. A promising area for future theoretical research should investigate, in addition to the incentives to coalesce into new parties, how factions' incentives to split change as a function of electoral volatility and party organizational choices. Lastly, in this paper mergers are assumed to be stable alliances. It would be interesting to characterize conditions under which, once they occur, mergers are stable and when we instead observe cycles of splits and mergers.

References

Austen-Smith, David and Jeffrey Banks. 1990. "Stable Governments and the Allocation of Policy Portfolios." *American Political Science Review* 84(3):891–906.

Bandyopadhyay, Siddhartha, Kalyan Chatterjee and Tomas Sjostrom. 2011. "Pre-electoral Coalitions and Post-Election Bargaining." *Quarterly Journal of Political Science* 6(1):1–53.

Baron, David P and John A Ferejohn. 1989. "Bargaining in legislatures." *American Political Science Review* 83(4):1181–1206.

Bélanger, Éric and Jean-François Godbout. 2010. "Why do Parties Merge? The Case of the Conservative Party of Canada." *Parliamentary Affairs* 63(1):41–65.

Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge University Press.

Downs, Anthony. 1957. *An Economic Theory of Democracy*. New York; Harper and Row.

Franklin, Mark N and Thomas T Mackie. 1983. "Familiarity and Inertia in the Formation of Governing Coalitions in Parliamentary Democracies." *British Journal of Political Science* 13(3):275–298.

Frey, Anderson, Gabriel López-Moctezuma and Sergio Montero. 2021. "Sleeping with the Enemy: Effective Representation under Dynamic Electoral Competition." *American Journal of Political Science* .

Golder, Sona Nadenichek. 2006. *The Logic of Pre-Electoral Coalition Formation*. Ohio State University Press.

Hortala-Vallve, Rafael, Jaakko Meriläinen and Janne Tukiainen. 2021. "Pre-Electoral Coalitions: Insights into the Creation of Political Parties." *Unpublished Manuscript* .

Hug, Simon. 2001. *Altering Party Systems: Strategic Behavior and the Emergence of New Political Parties in Western Democracies*. University of Michigan Press.

Ibenskas, Raimondas. 2016. "Marriages of Convenience: Explaining Party Mergers in Europe." *The Journal of Politics* 78(2):343–356.

Ibenskas, Raimondas and Allan Sikk. 2017. "Patterns of Party Change in Central and Eastern Europe, 1990–2015." *Party Politics* 23(1):43–54.

Ibenskas, Raimondas and Nicole Bolleyer. 2018. "Forms of inter-party cooperation: Electoral coalitions and party mergers." *East European Politics and Societies* 32(3):451–472.

Kitschelt, Herbert, Zdenka Mansfeldova, Radoslaw Markowski and Gabor Toka. 1999. *Post-Communist Party Systems: Competition, Representation, and Inter-Party Cooperation*. Cambridge University Press.

Koole, Ruud. 1994. "The Vulnerability of the Modern Cadre Party in the Netherlands." *How Parties Organize: Change and Adaptation in Party Organizations in Western Democracies* pp. 278–303.

Laver, Michael and Kenneth A Shepsle. 1990. "Coalitions and Cabinet Government." *American Political Science Review* 84(3):873–890.

Levy, Gilat. 2004. "A Model of Political Parties." *Journal of Economic theory* 115(2):250–277.

Lijphart, Arend. 1984. *Democracies: Patterns of Majoritarian and Consensus Government in Twenty-One Countries*. Yale University Press New Haven.

Lijphart, Arend. 2012. *Patterns of Democracy: Government Forms and Performance in Thirty-Six Countries*. Yale University Press.

Martin, Lanny W and Randolph T Stevenson. 2010. "The Conditional Impact of Incumbency on Government Formation." *American Political Science Review* pp. 503–518.

Mershon, Carol and Olga Shvetsova. 2013. "The Microfoundations of Party System Stability in Legislatures." *The Journal of Politics* 75(4):865–878.

Morelli, Massimo. 2004. "Party Formation and Policy Outcomes under Different Electoral Systems." *The Review of Economic Studies* 71(3):829–853.

Olsen, Jonathan. 2007. "The Merger of the PDS and WASG: From Eastern German Regional Party to National Radical Left Party?" *German Politics* 16(2):205–221.

Powell Jr, G Bingham. 2000. *Elections as Instruments of Democracy: Majoritarian and Proportional Visions*. Yale University Press.

Rakner, Lise, Lars Svåsand and Nixon S Khembo. 2007. "Fissions and Fusions, Foes and Friends: Party System Restructuring in Malawi in the 2004 General Elections." *Comparative Political Studies* 40(9):1112–1137.

Shin, Euncheol. 2019. "A Model of Pre-Electoral Coalition Formation." *Games and Economic Behavior* 118:463–485.

Shugart, Matthew Soberg and Rein Taagepera. 1989. *Seats and Votes*. New Haven.

Snyder, James M and Michael M Ting. 2002. "An Informational Rationale for Political Parties." *American Journal of Political Science* pp. 90–110.

Strom, Kaare. 1984. "Minority Governments in Parliamentary Democracies: The Rationality of Nonwinning Cabinet Solutions." *Comparative Political Studies* 17(2):199–227.

Tavits, Margit. 2008. "On the Linkage Between Electoral Volatility and Party System Instability in Central and Eastern Europe." *European Journal of Political Research* 47(5):537–555.

Supplementary Appendix to *Electoral Volatility and Pre-Electoral Alliances*

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Appendix A: Proofs of Main Results

Proof of Proposition 1. To compute party i's vote share from running alone ($V_{i,2}$) it suffices to identify the location of the voter who is indifferent between each pair of parties. Let $v_{\ell c,2}$ denote the ideal point of the voter who is indifferent between ℓ and c in $t = 2$, where $v_{\ell c,2}$ is located at $(z_\ell + z_c)/2$. The voter who is indifferent between c and r, denoted by $v_{cr,2}$, is defined analogously. Then, the vote share of ℓ is the CDF of the distribution of voters' ideal points evaluated at $v_{\ell c,2}$. Let $\xi \in [-a, a]$. Since voters' bliss points are uniformly distributed on $\mathcal{Z}$, ℓ's vote share is simply:

$$V_{\ell,2} = \frac{2a + z_\ell - 2\xi}{4a},$$

which depends on the realization of the shock to voters' preferences. A positive (negative) realization of the shock shifts voters' ideal policies to the right (left) thereby increasing the vote share of party r (ℓ) by $|\xi|$.[14] Similarly, $V_{c,2} = (1 - z_\ell)/4a = V_{c,1}$ and

$$V_{r,2} = 1 - V_{\ell,2} - V_{c,2} = \frac{1}{2} - \frac{1 - 2\xi}{4a}. \tag{A-2}$$

The vote share of a PEC formed in the second period is derived analogously. Let $V^{\text{pec}}_{\ell c,2}$ be the vote share of a PEC between ℓ and c in $t = 2$. Similarly to $V_{\ell,2}$ (A-1), the PEC's vote share is computed by finding the location of the voter who is indifferent between $z^{\text{pec}}_{\ell c}$ and z_r. That is, $V^{\text{pec}}_{\ell c,2}$ solves

$$\frac{1}{2} + \frac{z^{\text{pec}}_{\ell c} + z_r - 2\xi}{4a} - V^{\text{pec}}_{\ell c,2} = 0, \tag{A-3}$$

which produces

$$V^{\text{pec}}_{\ell c,2} = \frac{2a - 2\xi + \lambda z_\ell + 1}{4a}. \tag{A-4}$$

[14] Clearly, if $\xi > a$:

$$V_{c,2} = \frac{\frac{z_c + z_r}{2} + (a - \xi)}{2a}, \tag{A-1}$$

and $V_{r,2} = 1 - V_{c,2}$ (and analogously when $\xi < -a$).

Similarly, the vote share of a PEC between c and r is

$$V^{\text{pec}}_{cr,2} = \frac{2a - \lambda + 2\xi - z_\ell}{4a}. \tag{A-5}$$

Finally, recall that $z^m_{\ell c,2} = z^{\text{pec}}_{\ell c,2}$, which implies that the vote share of a merger formed in $t = 2$ is analogous to that of a PEC: i.e., $V^m_{\ell c,2} = V^{\text{pec}}_{\ell c,2}$ and $V^m_{cr,2} = V^{\text{pec}}_{cr,2}$.

In what follows I define threshold values of the shock realization that determine parties' equilibrium behavior in the second period.

Definition 1. *Let $\underline{\xi}(z_\ell)$ be the value of ξ such that ℓ's vote share $V_{\ell,2} > 1/2$ for $\xi < \underline{\xi}(z_\ell)$. It follows from the expression of $V_{l,2}$ (A-1) that $\underline{\xi} = \frac{z_\ell}{2}$.*

Similarly, let $\overline{\xi}(z_\ell)$ be the value of the shock realization such that r's vote share $V_{r,2} > 1/2$ for $\xi > \overline{\xi}(z_\ell)$. It follows from the expression of $V_{r,2}$ (A-2) that $\overline{\xi} = \frac{z_r}{2}$.

Let us first consider parties' decision when $\xi > \overline{\xi}$. When r has the majority of votes, by running alone, r can implement its preferred policy. Similarly, when $\xi < \underline{\xi}$ party ℓ runs alone and wins, hence the implemented policy is $\hat{x}_2 = z_\ell$. Hence, for $\xi < \underline{\xi}$ $(\xi > \overline{\xi})$ ℓ (r) rejects a PEC proposal from c and in equilibrium parties run alone in the second period.

When $\underline{\xi} < \xi < \overline{\xi}$, no party obtains an absolute majority *if all parties run alone*, yet a party that runs alone against a PEC could obtain a majority of votes. In particular, when parties form PECs, it could be that (i) $V^{\text{pec}}_{\ell c,2} > 1/2$, (ii) $V^{\text{pec}}_{cr,2} > 1/2$, or both. The following definition derives values of the shock realization that define each of these occurrences.

Definition 2. *Let $\underline{\xi}^{pec}(z_\ell)$ be the value of ξ such that $V^{pec}_{cr,2} > 1/2$ for $\xi > \underline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{cr,2}$ (B-5) that*

$$\underline{\xi}^{pec} = \frac{z_\ell + \lambda}{2}. \tag{A-6}$$

Similarly, let $\overline{\xi}^{pec}(z_\ell)$ be the value of ξ such that ℓ's vote share $V^{pec}_{\ell c,2} > 1/2$ for $\xi < \overline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{\ell c,2}$ (B-4) that

$$\overline{\xi}^{pec} = \frac{1 + \lambda z_\ell}{2} \tag{A-7}$$

Let us analyze c's decision when $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$. Definition 4 implies that for these values of the shock realization both PECs would reach an absolute majority. Then, c's proposal determines which PEC is formed in equilibrium. Under the assumptions, both ℓ and r accept

c's proposal—as running alone would result in a certain loss—and in $t = 2$ a PEC is formed. Then, c's decision determines whether the PEC is between ℓ and c or between c and r. Running alone is strictly dominated for c, because it would result in the adoption of the policy preferred by the party with the plurality of votes. Party c compares the payoff from forming a PEC with ℓ with that of a PEC with r: since z_ℓ is closer to c's bliss point and $\lambda > 0$, c strictly prefers to propose an alliance to ℓ.

Finally, it could be that only one PEC has the absolute majority of votes in the second period. Suppose that $V^{\text{pec}}_{\ell c,2} > 1/2 > V^{\text{pec}}_{cr,2}$, which implies $V_{\ell,2} > \max\{V_{c,2}, V_{r,2}\}$.[15] If c were to propose a PEC to ℓ, ℓ would reject because it could set its preferred platform by forming a minority government after elections.[16] Similarly, because ℓ has a relative majority, a PEC between c and r would not change the post-electoral policy set by ℓ. Hence, when only a PEC between ℓ and c reaches the absolute majority of votes, in equilibrium parties run alone and ℓ forms a minority government (the case such that $V^{\text{pec}}_{cr,2} > 1/2$ is analogous). □

Proof of Proposition 2. Denote by $U_{i,2}(\neg m_1)$ the expected second-period payoff of party i, *when no merger formed in the first period*. Proposition 1 allows us to express $U_{i,2}(\neg m_1)$ as a function of electoral volatility. By the uniform assumption of the shock, the probability of ξ falling below some threshold x is $\Pr\{\xi < x\} = \frac{1}{2} + \frac{\psi}{2}(x)$, hence the expected payoff from the second period is:

$$U_{i,2}(\neg m_1) = \left[\frac{1}{2} + \frac{\psi}{2}\left(\frac{z_\ell + \lambda}{2}\right)\right] u_i(z_\ell) + \frac{\psi(\lambda-1)(z_\ell - 1)}{4} u_i(z^{\text{pec}}_{\ell c}) + \left[\frac{1}{2} - \frac{\psi}{2}\left(\frac{1+\lambda z_\ell}{2}\right)\right] u_i(z_r). \tag{A-8}$$

When c has a plurality of votes, it compares the expected payoff from running alone:

$$U^{\text{al}}_c = u_c(z_c) + \delta U_{i,2}(\neg m_1), \tag{A-9}$$

to the expected payoff from merging with ℓ or r, which is derived next.

Let us first analyze what happens in the second period following a merger between ℓ and c in $t = 1$. Since the merger persists in $t = 2$, the analysis is straightforward. Let $\tilde{\xi}_\ell$ be the value

[15]It follows from Definition 4 that this is the case for $\underline{\xi} < \xi < \underline{\xi}^{\text{pec}}$.

[16]Notice that this would not be true if the realized policy was determined after parliamentary negotiations among all parties. Section 6 analyzes this possibility.

of the shock realization such that a merger between ℓ and c obtains half of the vote share. Given the assumptions:

$$\tilde{\xi}_\ell = \frac{(1 + z^m_{\ell c})}{2}.$$

Then, for $\xi < \tilde{\xi}_\ell$, the policy outcome is $\hat{x}_2 = z^m_{\ell c}$, otherwise it is $\hat{x}_2 = 1$. Similarly, suppose that a merger between c and r formed in $t = 1$. Let $\tilde{\xi}_r$ be the value of the shock realization such that a merger between c and r obtains half of the vote share, where $\tilde{\xi}_r = (z_\ell + z^m_{cr})/2$. For $\xi > \tilde{\xi}_r$, the policy outcome is $\hat{x}_2 = z^m_{cr}$, otherwise it is $\hat{x}_2 = z_\ell$.

Denote by $U_{i,2}(m_{\ell c,1})$ the expected second-period payoff of party i, *when a merger between ℓ and c formed in the first period*. We can express $U_{i,2}(m_{\ell c})$ as

$$U_{i,2}(m_{\ell c}) = \left[\frac{1}{2} + \frac{\psi}{4}\big(1 + z^m_{\ell c}\big)\right] u_i(z^m_{\ell c}) + \left[\frac{1}{2} - \frac{\psi}{4}\big(z^m_{\ell c} + 1\big)\right] u_i(1). \tag{A-10}$$

Similarly, the expected payoff of party i from a merger between c and r can be written as

$$U_{i,2}(m_{cr}) = \left[\frac{1}{2} - \frac{\psi}{4}\big(z_\ell + z^m_{cr}\big)\right] u_i(z^m_{cr}) + \left[\frac{1}{2} + \frac{\psi}{4}\big(z_\ell + z^m_{cr}\big)\right] u_i(z_\ell). \tag{A-11}$$

Given the expressions (A-10-A-11), we can easily compare party c's expected payoff from merging with ℓ and r. The expected payoff of party i from a merger between ℓ and c is

$$U^m_{i,\ell c} = u_i(z^m_{\ell c}) + \delta U_{i,2}(m_{\ell c}), \tag{A-12}$$

where the realized policy in the first period coincides with the merged party's platform, since the merger has the majority of votes in $t = 1$. The expression for $U^m_{i,cr}$ is analogous.

Differentiating $U^{\text{al}}_c(\psi) - U^m_{c,\ell c}$ with respect to ψ yields:

$$\frac{\partial(U^{\text{al}}_c - U^m_{c,\ell c})}{\partial\psi} = \frac{a(4\psi + z_\ell(\lambda - \psi - 1)(\lambda + 3\psi - 1)) - \delta(z_\ell - 1)\left(z_\ell\left(\lambda + (\lambda - 1)\lambda^2 z_\ell + z_\ell + 1\right) + 1\right)}{4}$$

which is always positive under the assumptions: as volatility goes down (ψ goes up), the payoff from running alone increases. Let $\hat{\psi}^a$ be the value of volatility such that $U^{\text{al}}_c(\psi) - U^m_{c,\ell c}(\psi) = 0$. It is easy to show that a real root that solves $U^{\text{al}}_c(\psi) = U^m_{c,\ell c}(\psi)$ exists. The expression is lengthy and therefore omitted. It follows from the previous step of the proof that c runs alone for $\psi > \hat{\psi}^a$ and prefers to form a merger with the closest party ℓ for $\psi < \hat{\psi}^a$.

Let $\Delta_c^m(\psi) = U_{i,\ell c}^m - U_{i,cr}^m$. Differentiating $\Delta_c^{\mathrm{m}}(\psi)$ with respect to ψ yields

$$\frac{\partial \Delta_c^{\mathrm{m}}(\psi)}{\partial \psi} = -\frac{1}{4}\delta(\lambda-1)\left(\lambda^2+\lambda+\left(\lambda^2+\lambda+1\right)z_\ell^3+\lambda z_\ell^2+\lambda z_\ell+1\right), \tag{A-13}$$

which is always negative.

Let $\tilde{\psi}$ be the value of ψ such that $U_{c,\ell c}^m = U_{c,cr}^m$. Solving for ψ produces

$$\tilde{\psi} = \frac{2(z_\ell-1)\left(\delta-(\delta+2)\lambda^2\right)}{\delta(\lambda-1)\left(\lambda^2+\lambda+\left(\lambda^2+\lambda+1\right)z_\ell^2-\left(\lambda^2+1\right)z_\ell+1\right)}. \tag{A-14}$$

It follows from the previous step of the proof that $U_{c,cr}^m > U_{c,\ell c}^m$ for $\psi < \tilde{\psi}$. □

Proof of Remark 2. Differentiating $\Delta_c^{\mathrm{m}}(\psi)$ with respect to z_ℓ yields

$$\frac{1}{4}\left(-\delta(\lambda-1)\lambda\psi-3\delta\left(\lambda^3-1\right)\psi z_\ell^2-2z_\ell\left(\delta(\lambda-1)(\lambda(\psi+2)+2)+4\lambda^2\right)\right). \tag{A-15}$$

It is straightforward to show that (A-15) is positive for $\lambda > 1/2$, and negative for $\lambda < 1/2$.

Differentiating $\Delta_c^{\mathrm{m}}/\psi$ with respect to z_ℓ yields:

$$-\frac{1}{4}\delta(\lambda-1)\left(\lambda+3\left(\lambda^2+\lambda+1\right)z_\ell^2+2\lambda z_\ell\right), \tag{A-16}$$

which is always positive. □

Proof of Lemma 1. The expected payoff of party i from a PEC between ℓ and c is:

$$U_{i,\ell c}^{\text{pec}} = u_i(z_{\ell c}^{\text{pec}}) + \delta U_{i,2}(\neg m_1), \tag{A-17}$$

where the second component of the RHS is party i's expected payoff in $t=2$ when no merger is formed in $t=1$. The expressions for $U_{i,cr}^{\text{pec}}$ is analogous, with $u_i(z_{cr}^{\text{pec}})$ as first-period payoff.

The difference $U_{c,\ell c}^m - U_{c,\ell c}^{\text{pec}}$ simplifies to:

$$U_{c,\ell c}^m - U_{c,\ell c}^{\text{pec}} = -\frac{\delta\left(\lambda^2-1\right)z_\ell^2(\lambda\psi+\psi z_\ell+2)}{4}. \tag{A-18}$$

Differentiating (A-18) with respect to ψ produces:

$$-\frac{1}{4}\delta\left(\lambda^2-1\right)z_\ell^2(\lambda+z_\ell), \tag{A-19}$$

which is always negative.

Let $\hat{\psi}$ be the value of ψ such that $U^m_{c,\ell c}=U^{\text{pec}}_{c,\ell c}$, where

$$\hat{\psi}=-\frac{2}{\lambda+z_\ell}. \tag{A-20}$$

It follows from the first step of the proof that for $\psi>\hat{\psi}$, $U^{\text{pec}}_{c,\ell c}>U^m_{c,\ell c}$. $\square$

Proof of Proposition 3. For c to prefer a merger with ℓ, it must be that (i) $U^m_{c,\ell c}>U^m_{c,cr}$, (ii) $U^m_{c,\ell c}>U^{\text{pec}}_{c,\ell c}$, (iii) $U^m_{c,\ell c}>U^{\text{pec}}_{c,cr}$ and (iv) $U^m_{c,\ell c}>U^{\text{al}}_c$. Note that we can immediately compare the expected payoff from the two PECs, because the second period payoff is the same for both of them (B-11). This leads to the following strict ranking for party c: $U^{\text{pec}}_{c,\ell c}>U^{\text{pec}}_{c,cr}$, which simply follows from comparing the first-period payoffs. Clearly, conditions (i)-(iv) are necessary but not sufficient for a merger between c and ℓ to form in equilibrium, as the merger must be incentive compatible for ℓ as well. It is straightforward to derive similar rankings for ℓ and r respectively: $U^{\text{pec}}_{\ell,\ell c}>U^{\text{pec}}_{\ell,cr}$ and $U^{\text{pec}}_{r,cr}>U^{\text{pec}}_{r,\ell c}$.

Consider the following equilibrium candidate: c proposes a PEC to ℓ for $\psi>\hat{\psi}$, and a merger to ℓ for $\psi<\hat{\psi}$, where $\hat{\psi}$ is defined in (A-20). Party ℓ accepts a PEC proposal for $\psi>\hat{\psi}$ because it prefers a PEC with c to the alternatives from rejection: if ℓ rejects c's offer, c proposes a PEC to r, which always accepts. Recall that no party has an absolute majority of votes, which implies that the PEC with policy z^{pec}_{cr} would win against party ℓ. Since $z^{\text{pec}}_{\ell c}\succ_\ell z^{\text{pec}}_{cr}$ for all $\lambda>0$, ℓ accepts c's proposal for $\psi>\hat{\psi}$, and in equilibrium a PEC between c and ℓ forms in $t=1$.

For $\psi<\hat{\psi}$, c prefers to form a merger rather than a PEC with ℓ. Suppose that for $\psi<\hat{\psi}$ c proposes a merger to ℓ. If ℓ accepts, its expected payoff is $U^m_{\ell,\ell c}$. By assumption, c cannot propose a merger to r following a rejection of ℓ. Thus, if ℓ rejects, it receives a PEC proposal in the next period. Since $U^{\text{pec}}_{\ell,\ell c} > U^m_{\ell,\ell c}$, ℓ rejects the proposal, and in equilibrium a PEC between c and ℓ forms.[17]

We are left to check whether a merger between c and r can form for some ψ. From the proof of Proposition 2, we have that $U^m_{c,cr} > U^m_{c,\ell c}$ for $\psi < \tilde{\psi}$. Suppose that, for $\psi < \tilde{\psi}$, c proposes a merger to the extreme party r. Rejecting is strictly dominated for r, since the difference

$$U^m_{r,cr} - U^{\text{pec}}_{r,\ell c} = -(\lambda-1)^2 + \frac{1}{4}\delta(\lambda-1)^2(\lambda\psi + \psi z_\ell - 2) + \frac{1}{4}\delta(\lambda-1)\psi(z_\ell-1)(\lambda z_\ell - 1)^2 + (\lambda z_\ell - 1)^2$$

is positive under the assumptions. Hence, r always accepts a merger proposal from c. From the previous step of the proof it follows that in equilibrium a merger between c and r forms for $\psi < \tilde{\psi}$ and a PEC between c and ℓ forms for $\psi > \tilde{\psi}$. □

Proof of Proposition 4. To begin, consider parties' incentives to form alliances in the second period. As in the baseline model, if either ℓ or r have a majority, parties run alone in equilibrium. Suppose instead that ℓ has plurality. In this case, in equilibrium c proposes $\lambda^* = 0$ to ℓ, and ℓ accepts. Suppose this is not the case, and that ℓ rejects. Then, c would propose $\lambda^* = 0$ to r, and r would accept because the alternative would be z_ℓ implemented by ℓ. Party ℓ is indifferent between $\lambda^* = 0$ and the alternative from rejection, i.e., a PEC between c and r with platform z_c. As a tie-breaking rule, I assume that when indifferent parties join the alliance. Finally, because r always accepts a PEC proposal $\lambda = 0$, c is indifferent between the two PECs. I assume that when indifferent c proposes a PEC to the closest party ℓ.

It follows that the expected payoff from the second period is (given the uniformity of the shock):

$$U_{i,2}(\neg m_1) = \left[\frac{1}{2} + \frac{\psi}{2}\left(\frac{z_c + z_\ell}{2}\right)\right] u_i(z_\ell) + \frac{\psi}{4}(1 - z_\ell)u(z_c) + \left[\frac{1}{2} - \frac{\psi}{2}\left(\frac{z_c + z_r}{2}\right)\right] u_i(z_r). \quad \text{(A-21)}$$

[17] If instead we were to allow for sequential merger proposals, the equilibrium outcome would depend on c's ranking of alternatives, which varies with ψ. That is, suppose that $U^m_{c,\ell c} > U^{\text{pec}}_{c,\ell c} > U^m_{c,cr}$, Then, ℓ rejects the proposal, and in equilibrium a PEC between c and ℓ forms. If instead $U^m_{c,\ell c} > U^m_{c,cr} > U^{\text{pec}}_{c,\ell c}$, ℓ knows that a merger between r and c (its least preferred option) would form following a rejection. In the latter case, ℓ accepts c's offer and a merger between c and ℓ forms. Notice though that, even assuming sequential merger proposals, the parameter space for which mergers emerge in equilibrium is the same; the only difference is which party c merges with.

We can now analyze parties' decision to merge in the first period. From Proposition 2 and 3, we know that there exists a threshold value of volatility ($\tilde{\psi}$) that makes c indifferent between merging with ℓ and r, and that for $\psi < \tilde{\psi}$ ($\psi > \tilde{\psi}$) c prefers to merge with r (ℓ). The difference $U_c^{\text{al}} - U_{c,lc}^m$ simplifies to:

$$\frac{z_\ell \left(\delta\lambda\psi - \delta\left(\lambda^3 - 1\right)\psi z_\ell^2 - \delta z_\ell\left(\lambda^2(\psi+2)+2\right) - 4\lambda^2 z_\ell\right)}{4}. \tag{A-22}$$

Differentiating (A-22) with respect to ψ yields

$$\frac{\partial(U_c^{\text{al}} - U_{c,lc}^m)}{\partial\psi} = \frac{\delta z_\ell\left(\lambda - \left(\left(\lambda^3-1\right)z_\ell^2\right) - \lambda^2 z_\ell\right)}{4}, \tag{A-23}$$

which is always positive under the assumptions. Furthermore, there exists a threshold value of volatility ($\hat{\hat{\psi}}$) such that c is indifferent between running alone and merging with ℓ, where:

$$\hat{\hat{\psi}} = -\frac{2z_\ell\left(2\lambda^2 + \delta\left(\lambda^2-1\right)\right)}{\delta\left(\left(\lambda^3-1\right)z_\ell^2 + \lambda^2 z_\ell - \lambda\right)}. \tag{A-24}$$

I consider the same equilibrium of the baseline model as the candidate equilibrium: c proposes merger to r (ℓ) for $\psi < \tilde{\psi}$, a merger to ℓ for $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, and parties run alone for $\psi > \hat{\hat{\psi}}$.[18]

(i) Suppose $\tilde{\psi} < \psi < \hat{\hat{\psi}}$ (such that c prefers a merger with ℓ). Can c offer $\lambda = 0$ (i.e., its preferred policy)? Party ℓ's incentive compatibility condition for accepting a merger (when c has plurality) is:

$$\frac{1}{4}z_\ell\left(\delta\left(\left(-\lambda^3+2\lambda^2+1\right)\psi z_\ell^2 - \left((\psi+2)\lambda^2 - 4\lambda + 2\right)z_\ell + (\lambda-2)\psi + 4\right) - 4z_\ell(\lambda-2)\lambda\right) > 0. \tag{A-25}$$

Substituting $\lambda = 0$, The IC condition above simplifies to

$$\frac{1}{4}\delta z_\ell\left(-2\psi + \psi z_\ell^2 - 2z_\ell + 4\right) > 0, \tag{A-26}$$

[18]Notice that the value of $\tilde{\psi}$ is equal to the baseline model, whereas the value $\hat{\hat{\psi}}$ differs because of the different continuation value of not merging in $t = 1$.

where the sign of the LHS depends on the value of z_ℓ and ψ. In particular, offering $\lambda = 0$ is only incentive compatible for ℓ for high values of ψ, so that the LHS is positive. Are other values of λ sustainable? As λ increases, the merger platform shifts to the left, thus losing moderate votes to r. Furthermore, as the platform weight of ℓ increases, all else equal c prefers to run alone, thus c's incentive compatibility condition becomes more binding.

ii) Suppose $\psi < \tilde{\psi}$ (such that c prefers a merger with r). Can c offer $\lambda = 0$ to r? Party r's incentive compatibility condition for accepting a merger is:

$$\frac{1}{4}\delta\left(\lambda^3\psi - 2\lambda^2(\psi+1) + 4\lambda - \psi - (\lambda-2)\psi z_\ell^2 + z_\ell\left(\lambda^2\psi + 4\right) - 2\right) - (\lambda-2)\lambda > 0, \qquad \text{(A-27)}$$

which is not satisfied when $\lambda = 0$ for high electoral volatility ($\psi < \tilde{\psi}$), that is precisely when c would like to merge with r (rather than ℓ). Thus, for this range of volatility c needs to offer r a platform λ^* that makes r indifferent between merging and running alone. It is possible to show that such λ^* exists (the expression is lengthy and does not provide further intuition, and therefore I omitted it from the proof), and is such that $\lambda^* \in (0,1)$ for $\psi < \tilde{\psi}$. Finally, notice that:

$$\frac{\partial(U^m_{r,cr} - U^{\text{al}}_r)}{\partial\lambda} = -\frac{1}{4}(z_\ell - \lambda)(\delta(3\lambda\psi + \psi z_\ell - 4) - 8) < 0. \qquad \text{(A-28)}$$

This implies that any $\lambda > \lambda^*$ is accepted by r. For any such λ, an equilibrium where the centrist party merges with the extreme party r can be sustained. □

Proof of Proposition 5. I begin by showing that there are never static incentives to form PECs in a single period.

Consider c's choice in $t = 1$. The payoff from forming a PEC with ℓ is strictly greater than that of PEC with the more distant r. Nevertheless, the difference

$$U^{\text{al},1}_c - U^{\text{pec},1}_{c,\ell c} = \frac{(\lambda z_\ell + 1)^2(2a + \lambda z_\ell - 1)^2 - (z_\ell + 1)^2(2a + z_\ell - 1)^2}{16a^2} \qquad \text{(A-29)}$$

is positive for any $\lambda < 1$, thus c has no static incentives to form a coalition in $t = 1$.

Conversely, in $t = 2$ we have

$$U_c^{\text{al},2} = \int\limits_{-1/\psi}^{1/\psi} u_c\left(V_{\ell,2}z_\ell + V_{c,2}z_c + V_{r,2}z_r\right)\frac{\psi}{2}d\xi$$
$$= -\frac{\delta\left(3(z_\ell+1)^2(2a+z_\ell-1)^2 + \frac{4(z_\ell-1)^2}{\psi^2}\right) + 3(z_\ell+1)^2(2a+z_\ell-1)^2}{48a^2}, \qquad \text{(A-30)}$$

and the difference $U_c^{\text{al},2} - U_{c,\ell c}^{\text{pec},2}$ simplifies to

$$\frac{(2a(\lambda z_\ell+1)+(\lambda z_\ell-1)(-2\xi+\lambda z_\ell+1))^2 - (2a(z_\ell+1)+(z_\ell-1)(-2\xi+z_\ell+1))^2}{16a^2},$$

the sign of which depends on the value of ξ. In particular, for ξ low enough, c strictly prefers a PEC with ℓ to running alone. The same holds for a PEC with r, but the inequality is reversed. However, for those values of the shock neither ℓ nor r are willing to run. Hence, in a single-period game, no PECs emerge in equilibrium.

Let us now turn to parties' dynamic incentives. First, denote by $U_i(\neg m_1)$ party i's second period expected payoff if parties do not merge in $t = 1$. Since parties run alone in $t = 2$, for party c this is $U_c(\neg m_1) = U_c^{\text{al},2}$, and analogously for parties ℓ and r. Since PECs are statically dominated, party c compares $U_c^{\text{al}} = U_c^{\text{al},1} + \delta U_c^{\text{al},2}$ with $U_c(m_{\ell c,})$ and $U_c(m_{cr})$ in the first period, when deciding whether to propose a merger to any party.

Let $\Delta_c^{\ell c} = U_c(m_{\ell c}) - U_c^{\text{al}}$, where:

$$\Delta_c^{\ell c} = \frac{(\lambda-1)z_\ell\left(-3(\delta+1)\psi^2(2a+\lambda z_\ell+z_\ell)\left(2a(\lambda z_\ell+z_\ell+2)+\left(\lambda^2+1\right)z_\ell^2-2\right)-4\delta(\lambda z_\ell+z_\ell-2)\right)}{48a^2\psi^2}.$$

Differentiating with respect to ψ yields:

$$\frac{\partial \Delta_c^{\ell c}}{\partial \psi} = \frac{\delta(\lambda-1)z_\ell(\lambda z_\ell+z_\ell-2)}{6a^2\psi^3},$$

which is always negative under the assumptions. Furthermore, there exists a value $\hat{\psi}^{\ell c}$ such that $\Delta_c^{\ell c,2}(\hat{\psi}^{\ell c}) = 0$ (the expression is long therefore omitted). Hence, c prefers to merge with ℓ (run alone) for $\psi < \hat{\psi}^{\ell c}$ ($\psi > \hat{\psi}^{\ell c}$). Analogously, we have that $\partial\Delta_c^{cr}/\partial\psi < 0$ and that there exists

$\hat{\psi}^{cr}$ such that $\Delta_c^{cr,2}(\hat{\psi}^{cr}) = 0$. Hence, c prefers to merge with r for $\psi < \hat{\psi}^{cr}$, while it prefers the continuation value from a PEC for $\psi > \hat{\psi}^{cr}$.

It is left to show that a merger is not incentive compatible neither for ℓ nor for r. For ℓ, we have:

$$\Delta_\ell^{\ell c,2} = \frac{\begin{array}{c} z_\ell\Big(-3(\delta+1)\left(\lambda^2-1\right) z_\ell\left(\left(\lambda^2+1\right) z_\ell^2-2\right) - \frac{4\delta}{\psi^2}(\lambda-1)(\lambda z_\ell+z_\ell-2)\Big) \\ -12a(\delta+1)\left(-\lambda+z_\ell\left(\lambda^2+\left((\lambda-2)\lambda^2-1\right) z_\ell-1\right)+3\right)-12a^2(\delta+1)(\lambda-3)((\lambda-1)z_\ell+2) \end{array}}{48a^2},$$

which is always negative under the assumptions. It follows that ℓ rejects a merger proposal by c. Similarly, $\Delta_r^{cr,2} < 0$, and r rejects a merger proposal by c.

Since mergers are always dominated for both ℓ and r, in equilibrium no alliance forms in $t = 1$, and the unique equilibrium for all parameter values is that all parties run alone. □

Appendix B: Coalition Platforms as a function of Vote Shares

Proof of Proposition 1

Denote by $V_{i,t}$ party i's vote share at time $t = \{1, 2\}$, where $V_{i,1} < 1/2$ for each party i. Suppose that ℓ and c merge or form a PEC in t. Then, the policy platform of the resulting party or PEC in t is a convex combination of the constituent parties' bliss points:

$$z_{lc,t}^{m} = z_{lc,t}^{\text{pec}} = \lambda_{l,t}\, z_\ell + (1-\lambda_{l,t})\, z_c. \tag{B-1}$$

The weight $\lambda_{l,t} \in (0, 1)$ measures the relative electoral strength of the extreme party (ℓ) in t, which depends on the parties' vote shares as follows:

$$\lambda_{l,t} = \frac{1}{2} + \phi(V_{l,t} - V_{c,t}), \tag{B-2}$$

where the parameter $\phi \in \mathbb{R}_+$ is a normalization ensuring that $\lambda_{l,t} \in (0, 1)$. Equation B-1 implies that the policies resulting from PECs and mergers are equivalent *only in the same period*. Because of the electoral shock, the policy resulting from a merger (or PEC) formed in $t = 2$ is different from the policy resulting from a merger formed in $t = 1$ and persisting in $t = 2$. This is because the shock changes parties' relative vote shares and in turn the weight each party has in the common platform. Crucially, while mergers "solidify" the relative power parties have in $t = 1$—which is given by each party's vote share $V_{i,1}$—PECs are re-negotiated in $t = 2$, allowing parties to be flexible to changes in the electoral environment.

The analysis largely proceeds as in the baseline model. In what follows, I will highlight the differences that emerge from having platforms dependent on vote shares. I begin with the second period. Let $V^{\text{pec}}_{lc,2}$ be the vote share of a PEC between ℓ and c in $t = 2$. Similarly to $V_{l,2}$ (A-1), the PEC's vote share is computed by finding the location of the voter who is indifferent between $z^{\text{pec}}_{lc,2} = \lambda_{l,2}\, z_\ell + (1 - \lambda_{l,2})\, z_c$ and $z_{r,2}$. That is, $V^{\text{pec}}_{lc,2}$ solves

$$\frac{1}{2} + \frac{z^{\text{pec}}_{lc,2}(V^{\text{pec}}_{lc,2}) + z_r - 2\xi}{4a} - V^{\text{pec}}_{lc,2} = 0, \tag{B-3}$$

which produces

$$V^{\text{pec}}_{lc,2} = \frac{8a^2 + 2a(-4\xi + z_\ell\phi + z_\ell + 2) + z_\ell\phi(-2\xi + 2z_\ell - 1)}{16a^2}. \tag{B-4}$$

Similarly, the vote share of a PEC between c and r is

$$V^{\text{pec}}_{cr,2} = \frac{8a^2 - 2a(-4\xi + 2z_\ell + \phi + 1) - \phi(2\xi + z_\ell - 2)}{16a^2}. \tag{B-5}$$

Recall that $z^{m}_{lc,2} = z^{\text{pec}}_{lc,2}$ (B-1), which implies $V^{m}_{lc,2} = V^{\text{pec}}_{lc,2}$ and $V^{m}_{cr,2} = V^{\text{pec}}_{cr,2}$. What determines parties' choice in the second period? The shock has a twofold impact on parties' decision: first, it has a *direct* effect on parties' vote share, by swinging voters' preferences in favor of either ℓ or r. I denote this the *electoral effect*. Second, by changing parties' relative vote share, the shock *indirectly* affects parties' influence on the final policy of a PEC. I denote this the *policy effect*. In what follows I define threshold values of the shock realization that determine which of these two effects prevails in parties' decision to form a PEC in $t = 2$. These values also provide useful cutoffs to describe parties' equilibrium behavior in the second period.

Definition 3. *Let $\underline{\xi}(z_\ell)$ be the value of ξ such that ℓ's vote share $V_{l,2} > 1/2$ for $\xi < \underline{\xi}(z_\ell)$. It follows from the expression of $V_{l,2}$ (A-1) that $\underline{\xi} = \frac{z_l}{2}$.*

Similarly, let $\overline{\xi}(z_\ell)$ be the value of the shock realization such that r's vote share $V_{r,2} > 1/2$ for $\xi > \overline{\xi}(z_\ell)$. It follows from the expression of $V_{r,2}$ (A-2) that $\overline{\xi} = \frac{z_r}{2}$.

Let us first consider parties' decision when $\xi > \overline{\xi}$. When a party has the majority of votes, the electoral effect trumps every other consideration: by running alone, r can implement its preferred policy. Similarly, when $\xi < \underline{\xi}$ party ℓ runs alone and wins, hence the implemented policy is $\hat{x}_2 = z_\ell$. Hence, for $\xi < \underline{\xi}$ ($\xi > \overline{\xi}$) ℓ (r) rejects a PEC proposal from c and in equilibrium parties run alone in the second period.

When $\underline{\xi} < \xi < \overline{\xi}$, no party obtains an absolute majority *if all parties run alone*, yet a party that runs alone against a PEC could obtain a majority of votes. In particular, when parties form PECs, it could be that (i) $V^{\text{pec}}_{lc,2} > 1/2$, (ii) $V^{\text{pec}}_{cr,2} > 1/2$, or both. The following definition derives values of the shock realization that define each of these occurrences.

Definition 4. *Let $\underline{\xi}^{pec}(z_\ell)$ be the value of ξ such that $V^{pec}_{cr,2} > 1/2$ for $\xi > \underline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{cr,2}$ (B-5) that*

$$\underline{\xi}^{pec} = \frac{2a(2z_\ell + \phi + 1) + (z_\ell - 2)\phi}{8a - 2\phi}. \tag{B-6}$$

Similarly, let $\overline{\xi}^{pec}(z_\ell)$ be the value of ξ such that ℓ's vote share $V^{pec}_{lc,2} > 1/2$ for $\xi < \overline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{lc,2}$ (B-4) that

$$\overline{\xi}^{pec} = \frac{2a(z_\ell\phi + z_\ell + 2) + z_\ell(2z_\ell - 1)\phi}{8a + 2z_\ell\phi}. \tag{B-7}$$

Let us analyze c's decision when $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$. Definition 4 implies that for these values of the shock realization both PECs would reach an absolute majority. Then, c's proposal determines which PEC is formed in equilibrium. Under the assumptions, both ℓ and r accept c's proposal—as running alone would result in a certain loss—and in $t = 2$ a PEC is formed. Then, c's decision determines whether the PEC is between ℓ and c or between c and r.[19] c compares the payoff from forming a PEC with ℓ, i.e.,

$$u_c(z^{\text{pec}}_{lc,2}) = -\frac{z_\ell^2(2a(\phi + 1) + \phi(2z_\ell - 2\xi - 1))^2}{16a^2}, \tag{B-8}$$

[19]Running alone is strictly dominated for c, because it would result in the adoption of the policy preferred by the party with the plurality of votes.

with the payoff from forming a PEC with r

$$u_c(z_{cr,2}^{\text{pec}}) = -\frac{(2a(\phi+1) + \phi(2\xi + z_\ell - 2))^2}{16a^2}. \tag{B-9}$$

The following results show how c's decision changes with different values of the shock realization and with the location of parties' platforms. In particular, Lemma B-1 shows that, as voters' preferences shift in favor of r (ℓ), the centrist party prefers a coalition with ℓ (r). Lemma B-2 then shows that c prefers an alliance with the ideologically closest party when voters' preferences are stable (i.e., $\xi = 0$). Finally, Proposition B-1 characterizes the (second period) equilibrium alliance configuration based on the value of the shock realization.

Lemma B-1. Policy Effect. *Let $\Delta_c^{pec}(\xi) = u_c(z_{lc,2}^{pec}) - u_c(z_{cr,2}^{pec})$. $\Delta_c^{pec}(\xi)$ is strictly increasing in ξ.*

Proof. Let $\Delta_c^{\text{pec}}(\xi) = u_c(z_{lc,2}^{\text{pec}}) - u_c(z_{cr,2}^{\text{pec}})$, where

$$\Delta_c^{\text{pec}}(\xi) = \frac{(2a(\phi+1) + \phi(2\xi + z_\ell - 2))^2 - z_\ell^2(2a(\phi+1) + \phi(-2\xi + 2z_\ell - 1))^2}{16a^2}.$$

Differentiating Δ_c with respect to ξ yields

$$\frac{\partial \Delta_c}{\partial \xi} = \frac{\phi\left(2a\left(z_\ell^2+1\right)(\phi+1) + (z_\ell - 1)\phi\left(-2\xi + 2z_\ell^2 - 2\xi z_\ell + z_\ell + 2\right)\right)}{4a^2}$$

which is always positive. □

When $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$ both PECs obtain a majority if formed. When this is the case, Lemma B-1 shows that the policy effect determines c's proposal decision. To see why, suppose that the shock realization is such that c is indifferent between the two coalitions. Now, let the value of the shock realization increase. This increase leads to a higher (lower) vote share of party r (ℓ), which means that r (ℓ)'s preferred policy weighs more (less) in a PEC between c and r (ℓ). Then, ceteris paribus, c would prefer to form a PEC with ℓ. Conversely, a lower value of the shock makes a coalition with r more appealing. Similarly to bargaining games à la Baron and Ferejohn (1989) where the proposer joins the weaker coalition, this policy effect prevails whenever c could achieve a majority by forming a PEC with both parties (i.e., when $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$).

Whether c forms a PEC with ℓ or r ultimately depends on the location of the platform z_ℓ. When $\xi = 0$, c is indifferent between ℓ and r (i.e., $\Delta_c^{\text{pec}}(0) = 0$) when z_ℓ and z_r are equidistant from z_c, and prefers the closer ally otherwise, as the next result shows.

Lemma B-2. *$\Delta_c^{pec}(0)$ is strictly increasing in z_ℓ.*

Proof. Differentiating Δ_c with respect to z_ℓ yields

$$\frac{\partial \Delta_c}{\partial z_\ell} = \frac{-2a^2 z_\ell(\phi+1)^2 - a\left(6z_\ell^2 - 2z_\ell - 1\right)(\phi+1)\phi + \left(-4z_\ell^3 + 3z_\ell^2 - 1\right)\phi^2}{4a^2},$$

which is always positive. □

Since $z_\ell \in (0,1)$, a corollary of Lemma B-2 is that when $\xi = 0$ party c prefers a coalition with ℓ. Furthermore, Lemma B-1 implies that when the shock favors r, c continues to prefer an alliance with ℓ. The next definition derives the value of the shock realization, $\hat{\xi}$, such that party c is indifferent between proposing a PEC to ℓ or r (i.e., $\Delta_c^{\text{pec}}(\hat{\xi}) = 0$) for any z_ℓ.

Definition 5. *Let $\hat{\xi}(z_\ell)$ be the value of the shock realization such that $\Delta_c^{pec}(\hat{\xi}) = 0$. It follows from the expression of Δ_c^{pec} (B-8-B-9) that*

$$\hat{\xi} = \frac{a(z_\ell - 1)(\phi+1) + \left(z_\ell^2 - z_\ell + 1\right)\phi}{(z_\ell+1)\phi}. \tag{B-10}$$

It follows from Lemma B-1 that c prefers to form a PEC with ℓ (r) when $\xi > \hat{\xi}$ ($\xi < \hat{\xi}$). Whenever both PECs obtain the majority of votes ($\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$), the threshold $\hat{\xi}$ determines which of the two PECs form.

Finally, it could be that only one PEC has the absolute majority of votes in the second period. Suppose that $V_{lc,2}^{\text{pec}} > 1/2 > V_{cr,2}^{\text{pec}}$, which implies $V_{l,2} > \max\{V_{c,2}, V_{r,2}\}$.[20] If c were to propose a PEC to ℓ, ℓ would reject because it could set its preferred platform by forming a minority government after elections. Similarly, because ℓ has a relative majority, a PEC between c and r would not change the post-electoral policy set by ℓ. Hence, when only a PEC between ℓ and c reaches the absolute majority of votes, in equilibrium parties run alone and ℓ forms a minority government (the case such that $V_{cr,2}^{\text{pec}} > 1/2$ is analogous).

[20]It follows from Definition 4 that this is the case for $\underline{\xi} < \xi < \underline{\xi}^{\text{pec}}$.

The following proposition summarizes the last observation and the previous results without proof, showing when parties form alliances or run alone in the second period, when no mergers form in the first period.

Proposition B-1. Second-Period Policy Outcome. *If $V_{c,2} > V_{l,2}, V_{r,2}$, in equilibrium parties run alone in $t = 2$. Suppose that c has no plurality, and that no merger formed in $t = 1$. Then in $t = 2$ parties form PECs for intermediate realizations of the shock ξ, and compete alone for extreme ones. In particular, for $\underline{\xi}^{pec} < \xi < \hat{\xi}$ ($\hat{\xi} < \xi < \overline{\xi}^{pec}$), a PEC between c, r (c, ℓ) forms, and $\hat{x}_2 = z^{pec}_{cr,2}$ ($z^{pec}_{lc,2}$). Conversely, when $\xi < \underline{\xi}^{pec}$ ($\xi > \overline{\xi}^{pec}$), parties run alone and $\hat{x}_2 = z_\ell$ (z_r).*

Compared to the baseline model, Proposition B-1 also shows that the policy outcome is non-monotonic in the rightist party's relative popularity: that is, it could be that the implemented policy shifts to the right as ℓ becomes more popular. This is a consequence of the pivotality of the centrist party in choosing coalitions: as the policy cost of an alliance with ℓ increases, ceteris paribus c prefers the weaker party r.

I now turn to the first period analysis. Let $V_{c,1} > \max\{V_{l,1}, V_{r,1}\}$. In $t = 1$, c compares the expected payoff from running alone to that of forming a merger. Proposing a PEC is clearly dominated because $u_c(z_c) > u_c(z^{\text{pec}}_{lc,1}), u_c(z^{\text{pec}}_{cr,1})$. Denote by $U_{i,2}(\neg m_1)$ the expected second-period payoff of party i, *when no merger formed in the first period*. The expected payoff from the second period is:

$$U_{i,2}(\neg\, m_1) = \left[\frac{1}{2} + \frac{\psi}{2}\left(\underline{\xi}^{\text{pec}}\right)\right]\, u_i(z_\ell) + \left[\frac{\psi}{2}(\hat{\xi}) - \frac{\psi}{2}\left(\underline{\xi}^{\text{pec}}\right)\right] V_{i,2}(z^{\text{pec}}_{cr,2}) + \left[\frac{\psi}{2}\left(\overline{\xi}^{\text{pec}}\right) - \frac{\psi}{2}\left(\hat{\xi}\right)\right] V_{i,2}(z^{\text{pec}}_{lc,2}) + \left[\frac{1}{2} - \frac{\psi}{2}(\overline{\xi}^{\text{pec}})\right]\, u_i(z_r), \tag{B-11}$$

where $V_{i,2}(z^{\text{pec}}_{lc,2})$ is the expected payoff of party i from the LC coalition platform, which depends on the realization of the shock:

$$V_{i,2}(z^{\text{pec}}_{lc,2}) = \int_{\hat{\xi}}^{\overline{\xi}^{\text{pec}}} u_i\left(z^{\text{pec}}_{lc,2}\right) \frac{1}{\overline{\xi}^{\text{pec}} - \hat{\xi}}\, d\xi, \tag{B-12}$$

and analogously for $V_{i,2}(z^{\text{pec}}_{cr,2})$. When c has a plurality of votes, it compares the expected payoff from running alone, i.e. $U^{\text{alone}}_c = u_c(z_c) + \delta U_{i,2}(\neg\, m_1)$, to the expected payoff from merging with ℓ or r, which is as in the baseline model. Differently from the second-period, all the first-period analysis is equivalent to the baseline model, but the expressions are significantly more cumbersome. While I omit the expressions for space constraints, it can be shown that all the main results are unchanged (all the expressions and detailed proofs for the extensions are available upon request).

Appendix C: Introducing Uncertainty over Platforms' Location

While each party is associated with a particular policy (its "brand"), z_i, parties typically feature heterogeneous preferences inside them. This heterogeneity is crucial, as the policy platform that is chosen by each party in a given election might differ from its policy brand (or, in other words, parties cannot fully pre-commit to policies). This section formalizes this idea by introducing noise in the location of parties' platforms.

Let $x_{i,t}$ be the policy platform that is selected by party i in a given election. This platform corresponds to the realization of the random variable $X_{i,t} = z_i + \varepsilon$, where $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. The smaller ε, the sharpest the message of the party (i.e., the most informative the party brand). We can interpret the support of X_i as follows. Parties typically gather multiple candidates who are proponents of different issues, some of which might be very far from the party brand. Depending on which of these candidates wins the election, the party platform could differ from the ex-ante party brand.

When ℓ and c merge, the resulting platform is a random variable centered at $z^m_{\ell c,1}$, the convex combination of the constituent parties' bliss points:

$$X^m_{\ell c,1} = z^m_{\ell c,1} + \varepsilon^m, \tag{C-1}$$

where $\varepsilon^m \sim \mathcal{N}(0, \sigma^2_m)$, and

$$\sigma^2_m = \sigma^2 + \frac{|z_\ell - z_c|}{\gamma}. \tag{C-2}$$

By creating a new political entity, mergers decrease the informativeness of the constituent parties' brands: for any distinct pair of platforms z_ℓ and z_c, $\sigma^2_m > \sigma^2$ for any $\gamma \in \mathbb{R}_+$. The noise that arises from a merger is increasing in the distance between its constituent parties' bliss points: since voters expect candidates to be drawn from anywhere between z_c and z_ℓ, the uncertainty cost increases with the distance among platforms.[21] Furthermore, the noise is decreasing in γ: as $\gamma \to \infty$, $\sigma_m^2 \to \sigma^2$. As such, γ could be interpreted as the amount of trust between the merger's partners.[22] The merged party's brand $z^m_{\ell c,1}$ is a convex combination of the constituent parties' bliss points, as in the previous section: $z^m_{\ell c,1} = \lambda_{\ell,1} z_\ell + (1-\lambda_{\ell,1}) z_c$, where $\lambda_{\ell,1} = \frac{1}{2} + \phi\left(V_{\ell,1} - V_{c,1}\right)$.

Differently from mergers, PECs preserve the identity of different parties. Thus, when two parties form a PEC the noise term is the same as when parties run individually: $\varepsilon \sim \mathcal{N}(0, \sigma^2)$.

Because parties cannot pre-commit to policies, voters do not know the exact policy each party selects and suffer an uncertainty cost which is captured by the variance of X_i. Formally, voter v's expected payoff from party i's platform is

$$\begin{aligned} EU_v(X_i) &= \mathbb{E}\big[-\big(X_i - z_v\big)^2\big] \\ &= -\big(z_i - z_v\big)^2 - \sigma^2, \end{aligned} \tag{C-3}$$

where $z_i = \mathbb{E}[X_i]$ and $\sigma^2 = \mathrm{Var}[X_i]$.[23]

To compute each party's vote share when parties run alone, we need to identify the location of the indifferent voter for each pair of parties. Since σ^2 is constant across parties, we can focus on the comparison between pairs of party brands (z_ℓ, z_c and z_c, z_r), as in the baseline model. The same holds when evaluating a PEC's vote share.

The analysis changes when computing the vote share of a merger. Denote by $v^m_{lc,r,2}$ the voter who is indifferent between party r and a merger between ℓ and c in the second period.

[21]This assumption is supported by empirical evidence showing that mergers are more likely to form between ideologically close parties (Ibenskas, 2016), which suggests that parties anticipate the electoral cost of merging.

[22]When deciding to merge, a party faces the risk that the other partner would renege on the agreement by increasing its policy influence above the agreed at the time of the merger. While I leave it exogenous, it is reasonable to think γ to be positively correlated with the constituent parties' previous experience of governing together, which can reduce the uncertainty about partners' behavior (Franklin and Mackie, 1983; Martin and Stevenson, 2010).

[23]The second equality follows from $\mathrm{Var}[X_i] = \mathbb{E}[X_i^2] - \mathbb{E}[X_i]^2 = \sigma^2$, which allows to re-express $EU_v(X_i)$ as

$$\begin{aligned} EU_v(X_i) &= -\sigma^2 - \mathbb{E}[X_i]^2 + 2\mathbb{E}[X_i] z_v - z_v^2 \\ &= -\big(\mathbb{E}[X_i]^2 - 2\mathbb{E}[X_i] z_v + z_v^2\big) - \sigma^2. \end{aligned}$$

That is, $v^m_{lc,r,2}$ solves:

$$-\left(v^m_{lc,r,2} - z^m_{\ell c,2}\right)^2 - \frac{|z_\ell - z_c|}{\gamma} + \left(v^m_{lc,r,2} - z_{r,2}\right)^2 = 0. \tag{C-4}$$

From the indifference condition (C-4) it is clear that parties sacrifice at least some of their vote share when deciding to merge (vis-à-vis forming a PEC). This is because—when z_ℓ and z_c differ—voters pay an uncertainty cost when voting for a merged party. Despite this cost from merging, the next result shows that the trade-off identified in Proposition 3 holds, as long as the uncertainty cost associated to the merger is not too high.

Proposition C-1. Equilibrium with Electoral Uncertainty. *When γ is high enough, in equilibrium parties form mergers when electoral volatility is sufficiently high (low ψ), and PECs for low electoral volatility (high ψ). When γ is low, in equilibrium c forms a PEC with the closest party (ℓ).*

Proof. The analysis of $t = 2$ is analogous to the baseline model. First, suppose that no merger formed in $t = 1$. Because $\sigma^2_m > \sigma^2$, mergers are dominated in the second period, and both voters' and parties' decision are identical to the baseline.

Suppose instead that a merger between C and R formed in $t = 1$. By assumption, the merger persists and faces party L. Notice that the probability that the merged party gets the majority in $t = 2$ is $\Pr\{\xi > \tilde{\xi}_r\} = 1 - F(\tilde{\xi}_r)$ (the same as in the baseline), because the informational cost is only paid by voters in $t = 1$ when the merger is formed. Hence, the expected second period payoff from merging (A-11) is the same as in the baseline model.

In $t = 1$, policy uncertainty introduced by mergers changes how vote shares are computed. Let $v^m_{l,cr,2}$ denote the voter who is indifferent between voting for party L and for a merger among C and R. Formally, $v^m_{l,cr,2}$ solves

$$-\left(v^m_{l,cr,2} - z^m_{cr,2}\right)^2 - \frac{1}{\gamma} + \left(v^m_{l,cr,2} - z_{l,2}\right)^2 = 0. \tag{C-5}$$

Solving for the indifferent voter yields:

$$v^m_{l,cr,2} = -\frac{4a^2\left(\gamma\left((\phi+1)^2 - 4z_\ell^2\right) + 4\right) + 4a\gamma(z_\ell - 2)\phi(\phi+1) + \gamma(z_\ell - 2)^2\phi^2}{8a\gamma(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}. \tag{C-6}$$

Using this expression, it is straightforward to compute the vote share of the merged party in $t = 1$:

$$V^m_{cr,1} = \frac{1}{2} + \frac{4a^2\left(\gamma\left((\phi+1)^2 - 4z_\ell^2\right) + 4\right) + 4a\gamma(z_\ell - 2)\phi(\phi+1) + \gamma(z_\ell - 2)^2\phi^2}{16a^2\gamma(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}. \tag{C-7}$$

Differentiating $V^m_{cr,1}$ with respect to γ yields

$$\frac{\partial V^m_{cr,1}}{\partial\gamma} = -\frac{1}{\gamma^2(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}, \tag{C-8}$$

which is always positive: as γ increases, the uncertainty paid by voter is reduced and the vote share of the merger increases.

Finally, we check if there exists a positive γ such that $V^m_{cr,1} = 1/2$. Solving for γ yields

$$\hat{\gamma} = \frac{16a^2}{4a^2\left(4z_\ell^2 - 1\right) - \phi^2(2a + z_\ell - 2)^2 - 4a\phi(2a + z_\ell - 2)}, \tag{C-9}$$

which is a positive real root. It follows that for $\gamma > \hat{\gamma}$, $V^m_{cr,1} > 1/2$ and the analysis is analogous to the proof of Proposition 3. In particular, let $\Delta_{c,cr} \equiv U^m_{c,cr} - U^{\text{pec}}_{c,cr}$, where

$$U^m_{c,cr} = -(z^m_{cr,1} - z_c)^2 - \sigma^2 - \frac{1}{\gamma} + \delta U_{i,2}(m_{cr}),$$

and

$$U^{\text{pec}}_{c,cr} = -(z^{\text{pec}}_{cr,1} - z_c)^2 - \sigma^2 + \delta U_{i,2}(\neg m).$$

Because uncertainty only affects $\Delta_{c,cr}$ via the term $1/\gamma$, it follows that $\partial(U^m_{c,cr} - U^{\text{pec}}_{c,cr})/\partial\psi$ is always negative, analogously to Equation A-19. Furthermore, for γ big enough, there exists a value of ψ such that $U^m_{c,cr} = U^{\text{pec}}_{c,cr}$, and the result in Proposition 3 continues to hold.

It is left to show that for γ small enough no mergers are sustainable in equilibrium. When $\gamma < \hat{\gamma}$, $V^m_{cr,1} < 1/2$. In this case we have

$$U^m_{c,cr} = -(z_l - z_c)^2 - \sigma^2 - \frac{1}{\gamma} + \delta U_{i,2}(m_{cr}).$$

Note that $U^m_{c,cr} \to -\infty$ as $\gamma \to 0$. This implies that there exists $\gamma^{'}$ small enough such that $\Delta_{c,cr}(\gamma^{'}) = 0$ has no solution. In particular, we have $U^{\text{pec}}_{c,cr}(\gamma^{'}) > U^m_{c,cr}(\gamma^{'})$ for all ψ. The analysis for a merger between C and L is analogous therefore omitted. $\square$

Intuitively, Proposition C-1 shows that mergers are only sustainable if they don't introduce excessive uncertainty about where the party platform stands. This can be the case for example when the merged party has a clear statute which is credible given the constituent parties' histories. Low uncertainty can also be a reasonable assumption if constituent parties have been former allies or have had previous experience of governing together. Conversely, Proposition C-1 shows that when voters' uncertainty about the new political party is high, a merger is not a viable alternative to a PEC *even when the electorate is very volatile*.