Why do Parties Merge?

Electoral Volatility and Long-Term Coalitions

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Abstract

What brings competing parties to coalesce into new entities? I present a model of electoral competition in which parties can form alliances and decide how binding these should be. Parties face a dynamic trade-off between insuring themselves against significant shifts in public opinion and allowing flexibility to respond to future electoral changes. The model shows that more binding alliances such as mergers emerge in equilibrium when electoral volatility is high; instead, when voters are predictable (e.g., highly partisan), parties either run alone or form more flexible pre-electoral coalitions. When the electorate is sufficiently volatile, a risk-averse centrist party might prefer to merge with an ideologically extreme party than with a moderate one.

1. Introduction

On October 11, 1980, the three confessional Dutch parties (the Catholic People Party, the Anti-Revolutionary Party and the Christian Historical Union) merged into the Christian Democratic Appeal (CDA). The merger was considered the most significant event in the development of the Dutch party system during that era (Gladdish, 1991, p. 54). The fusion played a crucial role in eradicating the division between Protestants and Catholics in Dutch politics, and it substantially reduced fragmentation within the party system (Koole, 1994). While the benefits of the union were obvious for the smaller parties, the decision of the Catholic People Party (KVP) to merge appeared more perplexing, given that it was the largest and most influential party at the time.

One way to understand the KVP's choice is to focus on the phenomenon of electoral volatility which characterized the 1970s' 'Dutch era of the floating vote' (Pedersen, 1979). In sharp contrast with the stability characterizing the 1960s — when Dutch politics was based on four 'pillars': Protestant, Catholics, Socialists and Liberals (Lijphart, 1989) — the 1970s saw the political consequences of secularization and 'depillarization'. In short, religion had become less important as a determining factor of voting behavior (Van Mierlo, 1986). The changing preferences of Dutch voters, who became increasingly secular and unpredictable, created an opportunity to appeal to a broader range of ideologies. This made the fusion a shrewd political move, since the coalition allowed the parties to present a wider ideological spectrum to the electorate, catering to the shifting preferences of Dutch voters. By merging, the KVP could insure itself against voters' volatility.

Other examples similarly suggest that volatility of the electorate may prompt political elites to consider mergers. In France, a volatile electorate allowed the far-right candidate Jean Marie Le Pen to reach the second round of the 2002 presidential election. The unexpected event triggered turbulence, deep political upheaval and institutional change.[1] In response, The UMP was formed in September 2002 as a merger of several center-right parties under the leadership of President Jacques Chirac. The Italian political landscape completely changed in 2007, when mergers across the ideological spectrum effectively transformed the system into bipolarism, with two main competing electoral cartels. The mergers took place after a close election in 2006 which left the country under great uncertainty about future electoral results.[2] In the UK, opinion polls in 1986 indicated that all major parties led at various points,[3] and in 1988 the Social Democratic Party and the Liberal Party formally merged as the Social and Liberal Democratic Party.

In this paper, I argue that electoral uncertainty can generate systematic incentives for leaders of political parties to form short-term alliances or permanent mergers with other parties, and I investigate the conditions under which this is the case. To do so, I present a dynamic model of electoral competition in which voters' preferences can change over time, inducing uncertainty among politicians regarding their future support. In this setting, a merger represents an *insurance device* when the electorate's future choices are hard to predict. The model

[1]Cf. Daley, S. (2002) 'THE FRENCH SURPRISE: THE SHOCK; French Political Leaders Rally Around Chirac', New York Times, 23 April: https://www.nytimes.com/2002/04/23/world/the-french-surprise-the-shock-french-political-leaders-rally-around-chirac.html..

[2]Cf. 'Prodi claims win in knife-edge Italian election', Financial Times, 11 April 2006: https://www.ft.com/content/86971d62-c872-11da-b642-0000779e2340..

[3]Cf. Hughes, D. (2019) 'Volatile voting: Why the next General Election is going to be a shambles', Electoral Reform Society: https://www.electoral-reform.org.uk/volatile-voting-why-the-next-general-election-is-going-to-be-a-shambles/.

shows that parties can merge out of fear of losing popularity in the future, even though mergers carry present costs. By providing conditions under which we should expect party systems to consolidate via endogenous party competition, this paper informs our understanding of party system dynamics.

The model features a two-period electoral game between three parties. Parties have concave preferences over policy and derive positive utility from being in office. Each party is associated with a fixed policy platform, which can be changed by forming an alliance. That is, by building an alliance parties can commit to policies that differ from their preferred ones and that could not be credibly implemented otherwise. The platform resulting from an alliance is a convex combination of the constituent parties' platforms. In each period the centrist party decides whether to run alone (in which case a government is formed in post-electoral bargaining) or to propose an alliance to the leftist or rightist party: the proposal can be to form a long-term alliance (a merger) or a short-term alliance (a pre-electoral coalition, hereafter PEC).

A merger is a binding arrangement that solidifies the relative power (i.e., electoral weight) constituent parties have at a given point in time, and *persists across elections*. Conversely, PECs are only *temporary* alliances that need to be renegotiated in each period, and that allow parties to preserve their identities. This assumption captures the idea that PECs are easier to dissolve, while reversing a merger imposes significant costs.

I first ask when parties compete *independently* at election time. I show that the centrist party only runs alone if it has a plurality of votes. This is because, if parties run alone, a centrist party with a plurality of votes can form a minority government enjoying all the rents from office.[4] However, having a plurality of votes is not sufficient for parties to run alone in equilibrium. Parties are forward-looking, and between periods an exogenous shock to voter preferences might change parties' vote shares. The centrist party is willing to run alone only when it has a plurality of votes in the first period *and* it is sufficiently likely to maintain a plurality in the second period (i.e., when electoral volatility is low). Given that the centrist party is the pivotal actor, under these conditions no alliances form and parties run alone in equilibrium.

In contrast, if the centrist party lacks a plurality of votes it does not run alone. If parties run alone, the policy would be determined through post-electoral bargaining, resulting in a final policy that is worse than what the centrist party could get forming an alliance with its closest ally before the election. Hence, in this case the question is whether parties form *short-term* or *long-term* alliances. Parties face a *dynamic* trade-off: while mergers insure constituent parties against unfavorable shifts in the electorate's preferences, these long-term forms of alliances come at the cost of losing the opportunity to join more advantageous coalitions in the future. Conversely, short-term alliances such as PECs offer more flexibility in changing coalition partners across time.

Results show that the choice between mergers and PECs crucially depends on *electoral volatility* (i.e., the likelihood of large shifts in voter preferences). When electoral volatility is high enough, in equilibrium parties form stable alliances such as mergers. Intuitively, if voter preferences can shift significantly in one direction, the party that is advantaged from the shift can govern alone. Hence, high volatility poses a risk for the centrist party, which may be left

[4] This assumption follows the empirical regularity that minority governments tend to be formed by the dominant party (Laver and Benoit, 2015).

out of power. Conversely, as voter preferences become more stable, the centrist party faces a reduced risk of being excluded from power, and more flexibility of being able to choose an attractive coalition partner after an election becomes more important. Electoral instability is often considered a characteristic of the early years of democratic regimes (Kitschelt et al., 1999). This result provides an explanation for the empirical observation that the frequency of mergers decreases as democratic regimes mature (Ibenskas and Sikk, 2017).

Surprisingly, when the electorate is sufficiently volatile, the centrist party may prefer to merge with the *more extreme* party. Intuitively, the worst possible outcome for the centrist party is that the extremist becomes so popular to win an outright majority in the future. Since parties are risk-averse, the centrist outweighs this possibility and merges with the extremist, even when doing so means to forego the possibility to enjoy the rents from forming a single-party government in the current period. This result is robust to considering pre-electoral bargaining among constituent parties, where the merger platform is proposed by the centrist party.

Most of the literature on party system stabilization focuses on voters as the main driver of stabilization, both in Western countries (Dalton and Flanagan, 2017; Pedersen, 1979; Shamir, 1984; Taagepera and Grofman, 2003) and in new democracies such as Africa (Kuenzi and Lambright, 2001), Latin America (Coppedge, 1998) and Eastern Europe (Birch, 2003). In these accounts parties are primarily by-products of pre-existing societal cleavages, and instability in party system results from instability in such cleavages or voter preferences (Tavits, 2008). In contrast, the model I propose highlights that elites who strategically respond to electoral volatility might also affect the stability of the party system. By focusing explicitly on the elite's strategic choice, this model contributes to the discussion on party system stabilization by showing that volatility does not necessarily lead to an unstable party system, and that under certain condition it can even lead to its consolidation with the creation of long-term alliances.

2. Contribution to the Literature

The literature on pre-electoral alliances has generally emphasized institutional and ideological factors that matter for alliance formation. Disproportional electoral institutions create economies of scale that incentivize the formation of pre-electoral alliances (Strom, Budge and Laver, 1994; Kaminski, 2001; Golder, 2006*a*,*b*; Clark and Golder, 2006; Blais and Indridason, 2007; Ibenskas, 2016*a*,*b*). In general, pre-electoral agreements are also more likely to form among ideologically similar parties (Golder, 2006*b*; Ibenskas, 2016*b*), although recent evidence from Mexican local elections shows that ideologically distant parties are willing to form alliances to remove entrenched incumbents from office (Frey, López-Moctezuma and Montero, 2021). Importantly, these contributions do not directly examine the conditions under which parties prefer short-term or long-term alliances—the focus of the current paper. Methodologically, I provide a *dynamic* theory that differentiates between mergers and PECs, which contributes to existing models of multiparty competition (Austen-Smith and Banks, 1988; Bandyopadhyay, Chatterjee and Sjostrom, 2011; Shin, 2019; Buisseret, 2017; Buisseret and Van Weelden, 2020). Substantively, the model suggests that electoral volatility can generate incentives for party mergers.

This paper contributes to the literature on endogenous party formation, reviewed in Dhillon (2005). In particular, Levy (2004) analyses party formation in the presence of a multidimensional policy space, where policy-motivated politicians can form coalitions (parties) to credibly commit to a broader set of policies (the Pareto set of the coalition). I model parties forming coalitions, and focus on their dynamic trade-off. Beside serving the role of *commitment* device, in my model stable coalitions (i.e., mergers) act as an *insurance* device against negative electoral outcomes. In Morelli (2004) as well, parties help voters to coordinate. He analyzes multi-district elections in a uni-dimensional policy space under different electoral rules. Snyder and Ting (2002) consider a fixed number of parties and analyze endogenous platforms, or brands, which allow candidates to signal their preferences to voters.

Finally, the paper contributes to the literature analyzing the determinants of party system dynamics (Dalton and Flanagan, 2017; Pedersen, 1979; Shamir, 1984; Taagepera and Grofman, 2003; Kuenzi and Lambright, 2001; Coppedge, 1998; Birch, 2003; Tavits, 2008). By providing a theory of party consolidation that emerges endogenously through parties' strategic choice, this paper joins several empirical studies that have recognized that party system stabilization is not just the product of voter demand factors (Cox, 1997; Kitschelt et al., 1999; Tavits, 2008). Here, the crucial contribution is to show that, through electoral competition, parties can induce stability in the party system even if (and precisely because) voters are volatile.

3. The Model

Consider a two-period game of electoral competition between three parties, indexed by i, where $i \in \{\ell, c, r\}$. Each period features a proposal stage, which determines parties' alliances, and an election. Each party is associated with a preferred policy platform $z_i \in \mathbb{R}$, where $z_\ell < z_c < z_r$. There exists a continuum of voters, indexed by v, who vote for one of the parties. Voters' ideal points are uniformly distributed over a subset of the policy space, $\mathcal{Z} \equiv [-a, a]$, where $\mathcal{Z} \subset \mathbb{R}$.[5] The ideal policy of voter v is denoted by $z_v \in \mathcal{Z}$.

Proposal Stage. In each period, the proposal stage unfolds as follows. First, the centrist party decides whether to run alone or not. If c runs alone, no further action is taken and parties compete independently by proposing their preferred platforms.[6] Second, if c does not choose to run alone it can propose to either ℓ or r to form a merger. If c's proposal to ℓ (r) is accepted, the merged party runs against r (ℓ). If c's proposal is rejected or if no merger is proposed, c can propose a PEC to either ℓ or r. If c's proposal to ℓ (r) is accepted, the PEC formed by ℓ, c (c, r) runs against r (ℓ). Otherwise, parties compete independently. Figure 1 summarizes the proposal stage sequence and the resulting configurations of alliances in the electoral stage when c makes a proposal to ℓ (the case of r is analogous).

Let me highlight the following features of the proposal stage. First, alliances between ideologically non-connected parties (ℓ and r) are ruled out. Second, parties form mergers before PECs. Empirically, while parties typically form PECs shortly before elections, mergers can occur at any point of a legislative term. While this protocol rules out merger proposals after PECs proposal, the results would be unchanged if mergers were proposed after PECs. As the analysis below clarifies, the only relevant assumption is that mergers take place in the first electoral period, before the realization of the shock to voter preferences.

[5] This assumption is without loss of generality and is merely convenient for computing parties' vote shares.

[6] Allowing parties to strategically announce their policy platforms does not affect the analysis, since platforms that differ from parties' bliss points would not be credibly implemented in equilibrium.

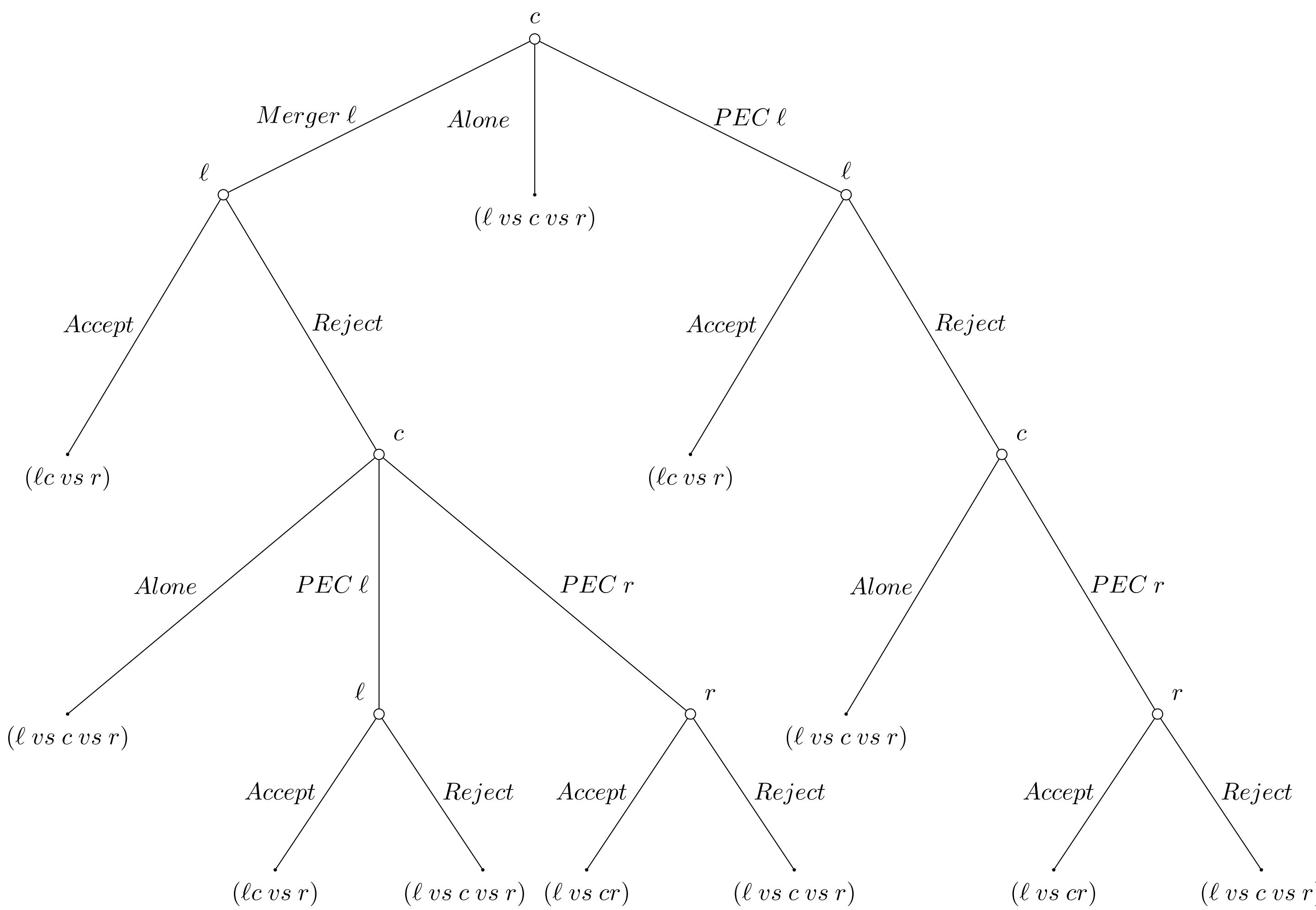

Figure 1 – Proposal Stage Sequence. The sequence includes c's proposal to ℓ (the case of r is analogous). The parentheses indicate the alliances' configurations at the electoral stage.

After the proposal stage is completed, an election takes place, resulting in the formation of a government and the adoption of a policy.

Government. I assume that seat shares are proportional to vote shares, or in other words that the electoral system is perfectly proportional.[7] Thus, in what follows I will focus on parties' vote shares. If parties compete alone and a party wins a majority of votes, it forms a

[7]For empirical evidence that electoral alliances are common in countries that use proportional representation see Golder (2005).

single-party government. If a PEC or a merger wins a majority of votes, it forms a government after the election.[8] If parties compete alone and the centrist party has a plurality of votes, c forms a minority government.[9] If no party/PEC/merger has a majority and c has no plurality, a post-electoral coalition government forms.

Implemented Policy. If a party, PEC or merger wins a majority of votes in t, the implemented policy $\hat{x}_t$ is the policy chosen by the winner of the election. The policy chosen by an alliance is a convex combination of the constituent parties' bliss points. Suppose that ℓ and c merge, or form a PEC. The resulting policy platform, denoted $z_{\ell c}^m$ and $z_{\ell c}^{\text{pec}}$ respectively, is equal to:

$$z_{\ell c} = \lambda z_\ell + (1 - \lambda) z_c, \tag{1}$$

where the weight $\lambda \in (0, 1)$ captures the relative influence of the extreme party on the common platform. The policies resulting from an alliance between c and r are defined analogously. To streamline the analysis, I assume that λ is exogenously given and fixed in the presentation of the model in the main body of the paper.[10] Furthermore, the weight λ does not depend on the type of alliance. While this element could be incorporated in the model, the goal is to understand how dynamic incentives affect different configurations of alliances. For this

[8]The assumption that pre-electoral alliances are binding commitment is in line with the literature (Golder, 2006*a*; Carroll and Cox, 2007). For tractability, I am focusing on a limiting case where PECs (mergers) imply post-electoral coalitions, but it is enough to assume that PECs (mergers) increase the probability that constituent parties get to form a government.

[9]This assumption is empirically relevant because minority governments are common, at least in Europe, and tend to include the dominant party (Laver and Benoit, 2015).

[10]In the Appendix, I analyze the case where the weights depend on parties' vote shares, and as such change over time. While some of the results are more nuanced, the main trade-off that emerges from the analysis is analogous to this setup, where expressions are substantially simpler. Another extension of the model endogenizes the weight λ as the outcome of pre-electoral bargaining between the constituent parties. In this case as well the main results remain qualitatively unchanged.

purpose, I choose to shut down potential channels due to a bargaining distinction between the two types of alliances.

If no party/merger/PEC obtains a majority, the implemented policy is determined post-electorally, thus is unknown before the election. To capture the ex-ante uncertainty about the post-electoral process, I assume that the implemented policy is decided by a majority formed by c and ℓ with some probability α, and by c and r with complementary probability $1-\alpha$:

$$\hat{x}_t = \alpha z_{\ell c} + (1-\alpha) z_{cr}, \tag{2}$$

where α can be interpreted as the relative strength of ℓ in the post-electoral negotiations (relative to r). While modeling post-electoral negotiations is beyond the scope of this model, Expression (2) captures in reduced form the equilibrium outcome of a post-electoral bargaining game between parties in the legislature.[11]

Electoral Volatility. At the beginning of the second period an exogenous shock ξ affects all voters equally, where ξ is uniformly distributed in $\left[-\frac{1}{\psi}, \frac{1}{\psi}\right]$. Thus, in the second period voters' ideal points are distributed over $[-a+\xi, a+\xi]$. The support of the shock represents electoral volatility: as ψ decreases, the support of the shock becomes larger, and electoral volatility increases. Conversely, as ψ increases, the support of the shock shrinks and the electoral outcome becomes more predictable.

The analysis assumes that in the first period no party has a majority of votes (and seats), and that either ℓ or c have a plurality of votes. Because party vote shares depend on voter

[11]Notice that when c forms a minority government it would still need to find a legislative majority to approve every policy, so $\hat{x}_t$ would still be equal to expression 2.

preferences and these change in the second period due to the shock, in the second period it could be that any party has a plurality or majority of votes.

Play in the second period depends on the organizational decision of the first period. If no merger formed in $t = 1$, the proposal and election stages of the second period take place. For ease of exposition, I assume that mergers persist in $t = 2$ after being formed in $t = 1$. That is, constituent parties cannot split in the period that follows the merger formation. This limiting case is equivalent to assuming that splitting after a merger is formed is infinitely costly. However, results would be qualitatively unchanged as long as the cost of unwinding a merger exceeds that of unwinding a PEC.

Payoffs and Strategies. Voters have quadratic preferences over policies. Voter v's realized payoff from the implemented policy $\hat{x}_t$ is defined as $u_v(\hat{x}_t) = -\big(z_v - \hat{x}_t\big)^2$. Parties care about policies and office rents (R). Suppose that party i forms a single-party government after the election (this happens if i has a majority or if it is the centrist party running alone with a plurality of votes). Then, i's payoff is

$$u_i(\hat{x}_t) = R - \big(z_i - \hat{x}_t\big)^2, \tag{3}$$

where $\hat{x}_t$ is the policy that is implemented after the election in period t. When instead a PEC or a merger forms, R is divided among coalition partners.[12]

I focus on subgame perfect equilibria in pure strategies. A pure strategy for party c defines c's decision to run alone, form a merger with ℓ (r), or form a PEC with ℓ (r) in $t = 1$, as well as

[12] I assume that rents are equally divided among coalition partners, but it is straightforward to extend the analysis to the case where the share of rents depends on parties' relative electoral strength.

in $t = 2$ conditional on no mergers forming in $t = 1$. For party ℓ (r) a pure strategy defines (i) an acceptance decision following a merger proposal in $t = 1$ and, conditional on no merger proposals being made, an acceptance decision following a PEC proposal to ℓ (r); (ii) if no mergers formed in $t = 1$, an acceptance decision following a merger proposal to ℓ (r) in $t = 2$ and, conditional on no merger proposals being made, an acceptance decision following a PEC proposal to ℓ (r). Voters are myopic, and since no voter is ever pivotal, I adopt the standard assumption that voters vote sincerely.[13] Parties maximize their expected payoff and evaluate the future according to a common discount factor $\delta \in (0, 1)$.

4. Results

To begin, consider the incentives to form alliances in the second period, when no merger formed in the first period. To simplify the notation, assume that $z_c = 0$, $z_r = 1$ and $z_\ell \in (-1, 0)$. Absent dynamic considerations, party c simply compares the payoffs from running alone and from forming an alliance. Denote by $V_{i,t}$ party i's vote share at time $t \in \{1, 2\}$, where $V_{i,1} < 1/2$ for each party i. Similarly, $V^{\text{pec}}_{\ell c,t}$ denotes the vote share of a PEC between ℓ and c at time t. Because $z^m_{\ell c} = z^{\text{pec}}_{\ell c}$ ($z^m_{cr} = z^{\text{pec}}_{cr}$), c is indifferent between merging and forming a PEC *in the second period*. I assume that, when indifferent, parties choose PECs over mergers.

There are two components of party c's payoff that change with its proposal decision. First, the rents from office. When the centrist party has a plurality ($V_{c,2} > \max\{V_{\ell,2}, V_{r,2}\}$), by running alone it obtains R, forming a minority government. When instead parties form a PEC, c needs to share rents with its coalition partner. The second component is the payoff from the

[13] Assuming forward-looking voters does not change the results, as long as the shock to voters' preferences between periods is zero in expectation.

implemented policy. When parties form a PEC, party c's payoff from the implemented policy is the payoff associated with the PEC's platform. When parties run alone, c obtains the payoff from policy $z_{\ell c}$ with probability α and from policy z_{cr} with probability $1-\alpha$.

Clearly, only when c has a plurality of votes there exists a trade-off between office and policy motives. In this case, party c compares the payoff from running alone to that of forming a PEC with ℓ. Formally, c runs alone if and only if:

$$\alpha u_c\left(z_{\ell c}\right)+(1-\alpha) u_c\left(z_{cr}\right)+\frac{R}{2}>u_c(z_{\ell c}), \tag{4}$$

which using Equation 1 simplifies to:

$$R>2(1-\alpha) \lambda^2\left(1-z_{\ell}^2\right). \tag{5}$$

Intuitively, when rents from office are high enough, c is willing to take the post-electoral policy gamble in order to enjoy full rents from office. Because λ is the bargaining weight attached to the extreme party (and is equivalent for ℓ and r), as λ increases, the risk posed by post-electoral bargaining for the centrist party increases because the weight of r in post-electoral policy negotiations increases. Hence, the centrist party is more eager to insure itself against r's influence post-electorally, and is only willing to run alone for higher office rents. Thus, as λ increases, condition 5 is only satisfied for higher office rents. Similarly, as α decreases, the likelihood of a post-electoral coalition with the more extreme party r increases. Hence, c's expected payoff from post-electoral negotiations decreases and c needs to obtain more office rents to compensate for the expected policy loss.

When can the centrist party successfully form a PEC in the second period? First, for a PEC to be incentive compatible *no party should have a majority of votes*, otherwise the majority party could implement its preferred policy both by running alone and by facing an opposing PEC. Since parties' vote share in $t = 2$ is affected by the shock ξ, a necessary condition for parties forming PECs in the second period is that the shock realization is not too extreme.

The following proposition shows when parties form alliances or run alone as a function of the shock realization, which determines the vote shares in the second period. The values of the shock realizations—which are derived in the Appendix— define for which vote shares PECs are incentive compatible for both constituent parties. In what follows we will assume that condition 5 holds.

Proposition 1. Second-Period Electoral Outcome. *Suppose that no merger formed in the first period. In the second period:*

1. *if* $\xi < \frac{z_\ell}{2}$*, parties run alone and* $\hat{x}_2 = z_\ell$*,*
2. *if* $\frac{z_\ell}{2} < \xi < a + z_\ell - \frac{1}{2}$*, a PEC between* c *and* ℓ *forms, and* $\hat{x}_2 = z_{\ell c}$
3. *if* $a + z_\ell - \frac{1}{2} < \xi < 1 - a - \frac{z_\ell}{2}$*, parties run alone and* $\hat{x}_2$ *is either* $z_{\ell c}$ *or* z_{cr}*,*
4. *if* $1 - a - \frac{z_\ell}{2} < \xi < \frac{1+\lambda z_\ell}{2}$*, a PEC between* c *and* ℓ *forms, and* $\hat{x}_2 = z_{\ell c}$*,*
5. *if* $\frac{1+\lambda z_\ell}{2} < \xi < \frac{z_r}{2}$*, parties run alone and* $\hat{x}_2$ *is either* $z_{\ell c}$ *or* z_{cr}*,*
6. *if* $\xi > \frac{z_r}{2}$*, parties run alone and* $\hat{x}_2 = z_r$*.*

Proof. All proofs can be found in the Appendix. □

Intuitively, for extreme values of the shock (cases 1 and 6) parties run alone because either ℓ or r have a majority and can form a government alone. Suppose that only a PEC between c and r would obtain a majority if formed (case 5), while a PEC between ℓ and c would lose against r. In this case, c compares the payoff from a PEC with the more distant party r to the payoff from running alone and engaging in post-electoral negotiations. The latter is preferred for any $\alpha \in (0, 1)$, hence for these values of the shock realization parties run alone.

When $V^{\text{pec}}_{\ell,c} > \frac{1}{2}$ (cases 2-4), a PEC between the closest party ℓ and c wins a majority of votes if formed. When this is the case, c compares the payoff from running alone to that of forming a PEC. When c does not have a plurality, c proposes a PEC to ℓ because of its closer platform (cases 2 and 4). This is because c prefers a PEC with ℓ to the lottery between $z_{\ell c}$ and z_{cr} after the election. When c has a plurality of votes (case 3), parties run alone and c forms a minority government after the election, since condition 5 holds (R is high enough).

First Period

We now turn to the first period. Similarly to the second period analysis, let us first consider whether the centrist party has a plurality of votes in the first period. If c is *not* the plurality winner and it decides to run alone, after the election it either forms a post-electoral government coalition with left or with right. In this case, the centrist party needs to divide the spoils with the coalition partner, and the policy is ex-ante (before the election) uncertain. If instead c forms a PEC (or a merger) with ℓ, it would still need to divide the spoils after the election, but it would suffer a lower expected policy cost by implementing the PEC policy with certainty. Hence in this case c never runs alone in equilibrium.

If c has a plurality, on the other hand, then it faces a trade-off. On the one hand, running alone allows c to enjoy all the rents from office in $t = 1$ because it forms a minority government. On the other hand, running alone entails a policy loss, because c faces a lottery over the policy outcome in the post-electoral phase. This is because the centrist party is uncertain about which party it will have to compromise with in the post-electoral phase. When condition 5 holds, office rents considerations trump the expected policy loss and c prefers to run alone.

In addition to this policy vs. office trade-off, there is a second key consideration in the first period. That is, the organizational choice of the first period influences parties' long-run prospects in the second period. This makes parties' strategic considerations in the first period more complex than the second period's ones, because in the second period parties only deal with the policy/office trade-off discussed in the previous paragraph. Indeed, if parties were myopic, in equilibrium c would run alone when having a plurality of votes for R sufficiently high. Yet, parties consider their expected second period payoff, which is affected by electoral volatility. The following analysis considers this trade-off and shows under what conditions parties run alone in equilibrium.

When do Parties Run Alone?

Suppose that c has a plurality of votes in the first period and condition 5 holds (i.e., R is sufficiently high). In this case, a PEC offers no advantage to c: the second period expected payoff from forming a PEC is identical to that of running alone because PECs are only temporary alliances, and running alone is preferred by c in the first period because office motives trump policy considerations. Therefore, c compares the expected payoff from forming a merger to that of running alone.

Recall that between the two elections, voters' preferences change. The magnitude of this change is crucial to determine whether c prefers to run alone or to merge with a different party. On the one hand, when volatility is low (ψ is high) c expects to keep its plurality status. While both ℓ and r prefer to join the centrist to set a more favorable policy, in this case c has an incentive to run alone, given the high chances of a future single-party government. When volatility is high (ψ is low), on the other hand, the likelihood that voters move away from the center increases, which in turn increases the incentives to merge as an insurance against an unfavorable change to voters' preferences.

To formalize this trade-off, we can express c's payoff from merging with ℓ as:

$$U^m_{c,\ell c} = u_c(z_{\ell c}) + \frac{R}{2} + \delta U_{c,2}(m_{\ell c}), \tag{6}$$

where the first two terms represent c's payoff in the first period, since c has to divide the spoils and the merged party can implement $z_{\ell c}$. By the uniform assumption of the shock, the probability of ξ falling below some threshold x is $\Pr\{\xi < x\} = \frac{1}{2} + \frac{\psi}{2}(x)$, hence the continuation value following a merger with ℓ in $t = 1$ is:

$$U_{c,2}(m_{\ell c}) = \left[\frac{1}{2} + \frac{\psi}{4}(1 + z_{\ell c})\right] \left(u_c(z_{\ell c}) + \frac{R}{2}\right) + \left[\frac{1}{2} - \frac{\psi}{4}(z_{\ell c} + 1)\right] u_c(1). \tag{7}$$

The expected payoff from merging with r ($U^m_{c,cr}$) is defined analogously.

Party c compares $U^m_{c,\ell c}$ and $U^m_{c,cr}$ to the expected payoff from running alone:

$$U_c^{\text{al}} = R + \alpha u_c\left(z_{\ell c}\right) + (1-\alpha)u_c\left(z_{cr}\right) + \delta U_{c,2}(\neg\, m_1), \tag{8}$$

where $U_{i,2}(\neg\, m_1)$ is the continuation value if parties do not merge in $t = 1$. This value depends on the shock realization, which determines the second-period electoral outcome as shown in Proposition 1.

Recall that $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. This assumption implies that we are stacking the deck *against* the emergence of mergers in equilibrium: mergers are more costly when c has a plurality of votes because of the need to forego a first-period single-party government where c gets all the spoils R. The next result shows that when the electorate's future choices are easily predictable, party c prefers to run alone and in equilibrium does not propose any alliance, whereas when volatility is high c prefers to merge, thus losing the opportunity to form a minority government and enjoy all the rents from office in the first period.

Suppose that volatility is sufficiently high so that c wants to merge in equilibrium. Which party does c prefer to merge with? The reader might expect that c always prefers a merger with the closer party ℓ. Indeed, the payoff that c obtains in the first period from merging with ℓ is higher than the one following a merger with r, because the implemented policy resulting from the former is closer to c's ideal point.

Surprisingly, this is not always the case: there are conditions under which c prefers merging with r. Specifically, Proposition 2 shows that when volatility is sufficiently high c prefers to merge with the ideologically more distant party (r).

Proposition 2. Equilibrium if c has a plurality. *Let $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. There exists a unique threshold $\hat{\psi}^a$ such that for $\psi > \hat{\psi}^a$ c prefers running alone to forming any merger. Furthermore, there exists a unique threshold $\tilde{\psi}$ such that for $\psi < \tilde{\psi}$ c prefers merging with r to merging with ℓ. In equilibrium:*

- *for $\psi > \hat{\psi}^a$, parties run alone and $\hat{x}_1$ is either $z_{\ell c}$ or z_{cr},*
- *for $\tilde{\psi} < \psi < \hat{\psi}^a$, a merger between c and ℓ forms, and $\hat{x}_1 = z_{\ell c}$,*
- *for $\psi < \tilde{\psi}$, a merger between c and r forms, and $\hat{x}_1 = z_{cr}$.*

The intuition behind this result is that a merger represents an insurance against an unfavorable outcome in the second period. The worst expected outcome for c is that the electorate moves to the right so that r obtains a majority of votes and $\hat{x}_2 = 1$, r's preferred platform. Since r is ex-ante disadvantaged, this event can only occur under high electoral volatility. If c were to merge with the closest party ℓ, the first period payoff would be higher than the payoff from merging with r. However, c would run the risk of losing against r in the second period, even if running as a merged party. Since preferences are concave, the latter policy cost following a victory of r carries more weight. As volatility increases, the worst case scenario becomes more likely, and c prefers to insure itself by merging with the more distant party r, thus giving up a first period solo rule and losing some moderate voters to ℓ.

The finding that the centrist party might form a merger with the more distant r to prevent its victory in the future might seem counter-intuitive. This result follows from the role of concavity and from the assumption that the platform implemented by the merged party—a convex combination of c and ℓ's bliss points —pulls some centrist voters towards the extreme party r. The result is also robust to an extension (see Appendix D) where c makes a take-or-leave offer $\lambda \in [0,1]$ to ℓ (r), thus showing how the qualitative result in Proposition 2 is unchanged when allowing for pre-electoral bargaining.[14]

Comparative Statics

An interesting question is how the trade-off between the two merger partners changes as their relative ideological extremism varies.

Remark 1. Ideological Extremism. *Let $\Delta_c^m(\psi) = U_{c,\ell c}^m - U_{c,cr}^m$ define party c's net expected payoff from merging with ℓ. Then,*

$$\frac{\partial^2 \Delta_c^m(\psi)}{\partial \psi \partial z_\ell} > 0.$$

From Proposition 2 we know that c's incentives to merge with the extreme party r increase with electoral volatility. Remark 1 shows that the magnitude of this incentive varies with the relative extremism of r: as z_ℓ moves towards zero, r becomes relatively more extreme. The positive cross-partial thus implies that an increase in volatility *expands* the region of the parameter space supporting an equilibrium where c merges with the extreme party more when r is relatively *more extreme*. Intuitively, when the extreme party is more of a threat for c, because the payoff associated to z_r is relatively more costly, then c is more prone to join it to prevent its solo victory in the future. Conversely, as z_ℓ moves away from the center, r becomes relatively more moderate and the advantage of merging with r vis-à-vis ℓ shrinks. Figure 2

[14]While parties' risk-aversion brings about mergers as an insurance against unfavorable electoral change, parties could merge even if risk-neutral. To see why, suppose that parties' preferences were linear in the policy component and that the shock to voters' preferences was expected to advance the right party. As long as the probability associated with a solo victory by r in the second period is sufficiently high, even a risk-neutral centrist party would merge in the first period. Thus, the logic of merging as insurance could also be applied to risk-neutral parties when the shock to voter preferences is expected to affect different parties asymmetrically.

provides an illustration of these results, plotting the region such that $\Delta_c^m(\psi) > 0$ as a function of ψ (x axis) and z_ℓ (y axis).

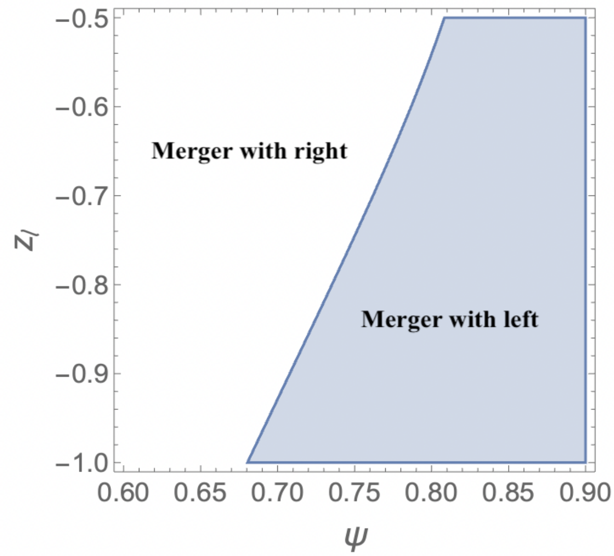

Figure 2 – Merger decision. $\Delta_c^{\mathrm{m}}(\psi)$ as a function of the value of ψ and z_ℓ. The blue region corresponds to the values of ψ, z_ℓ such that c prefers to merge with ℓ rather than with r ($\Delta_c^{\mathrm{m}} > 0$). For low values of ψ (i.e., high volatility), c prefers to merge with r, even if the latter is further away from c. As ℓ gets closer to c, c prefers a merger with r, ceteris paribus. The other parameters are set to $a = 1$, $\delta = 0.8$ and $\lambda = 0.4$.

Finally, Proposition 2 shows that office rents need to be sufficiently low for parties to merge. The next result shows that an increase in R makes office benefits more salient vis-à-vis policy considerations. As a result, the centrist party is more incentivized to run alone as the value of being in office increases.

Proposition 3. Office Benefits. *Let* $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. *For all parameter values:*

$$\frac{\partial \left(U_c^{al} - U_{c,\ell c}^m\right)}{\partial R} > 0.$$

As R increases, the centrist party benefits more from running alone than from merging, because it does not need to share R in the first period. Therefore, for R arbitrarily high c never merges in equilibrium, because it is simply too valuable to enjoy the benefits from forming a minority government in the first period.[15]

The Trade-off between Mergers and PECs

Suppose that c does not have a plurality of votes in the first period. What conditions can sustain an equilibrium in which parties merge in the first period? First, note that for c, forming a PEC with ℓ is always more advantageous than forming a PEC with r or running alone. In other words, either c prefers to merge with any of the other parties or to form a PEC with ℓ.

Which of the options c prefers — including the possibility of merging with r — depends on electoral volatility. On the one hand, the centrist party wants to insure itself against negative electoral shocks. Mergers provide such an insurance, by tying the centrist policy to a common platform which has higher chances to be implemented in the future. On the other hand, c values the flexibility provided by a PEC, which allows to form a coalition with either ℓ or r in the second period. Interestingly, this trade-off is still present even in the absence of office motives in the first period (c needs to share office rents both in case of a PEC and a merger), which distinguishes this case from the previous one where c foregoes merging to enjoy rents from a minority government.

The next result formalizes these observations, showing how the centrist party's incentives to merge change as a function of electoral volatility.

[15]Notice that I assumed that when $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$ the centrist party forms a minority government with certainty. However, it suffices to assume that the probability of c forming a minority government is higher when c has a plurality than it is otherwise.

Lemma 1. *Let $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$. There exists a unique $\hat{\psi}$ such that:*

- *for $\psi < \hat{\psi}$, c prefers to merge with ℓ,*
- *for $\psi > \hat{\psi}$, c prefers to form a PEC with ℓ.*

Lemma 1 shows that — even when the centrist party is not the plurality winner — high values of volatility call for mergers, while as the electorate becomes more stable the centrist party values more the flexibility provided by PECs.

We can now describe the equilibrium of the baseline game when c does not have a plurality of votes. The next result shows that different alliance configurations emerge in equilibrium, depending on electoral volatility. Recall that $\tilde{\psi}$ defines the value of ψ such that c is indifferent between merging with ℓ or r. We have:

Proposition 4. Equilibrium if c has no plurality. *Let $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$. In equilibrium:*

- *for $\psi < \tilde{\psi}$, a merger between c and r forms, and $\hat{x}_1 = z_{cr}$,*
- *for $\psi > \tilde{\psi}$, a PEC between ℓ and c forms, and $\hat{x}_1 = z_{\ell c}$.*

Proposition 4 provides a rationale for parties' incentives to join different types of alliances. When the likelihood of large shifts in voters' preferences is high enough, in equilibrium the centrist party prefers to merge rather than to form a PEC. By merging, the centrist party insures itself against large shifts in the electorate's preferences at the cost of losing the opportunity to form a more advantageous coalition in the future.

Interestingly, when the electorate is highly unpredictable ($\psi < \tilde{\psi}$), risk-aversion considerations trump the first period policy cost, resulting in a merger with the more distant party

r. As Proposition 2 shows, this happens because c wants to insure itself against the worst possible outcome (i.e., r implementing its bliss point).

Mergers are not sustainable anymore when voters' preferences are stable—which can be empirically associated with a highly partisan electorate. In this case, the centrist party values more flexibility, and forms with the closest party a temporary alliance which does not bind its policy platform in the future. By forming a PEC in the first period, the centrist party maintains its identity, preserving its brand for the future election, when more information about voters' preferences is available.[16]

Figure 3 provides a graphical representation of the equilibrium, illustrating which types of alliances (if any) emerge in equilibrium as a function of electoral volatility: the top panel assumes that $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$, while in the bottom one $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$. In the top panel, the dark gray region plots the range of parameters sustaining an equilibrium where c and r merge in the first period, while the light gray region plots the range for which ℓ and c form PECs in the first period, as a function of ψ (x axis) and parties' discount factor (y axis). Being electorally advantaged, ℓ always prefers a PEC to a merger with c. Hence, when volatility is low (ψ high enough), c proposes a PEC to ℓ, which accepts, and a PEC forms in equilibrium (light gray region). As electoral volatility increases (ψ decreases), the centrist party's incentives to merge increase, and a merger forms for $\psi < \tilde{\psi}$.

[16]Finally, note that the centrist party never merges with ℓ in equilibrium, even though for some values of volatility ($\tilde{\psi} < \psi < \hat{\psi}$) c would prefer to do so. This follows from the merger protocol assumption: ℓ would reject a merger proposal from c knowing that c is not able to propose a merger to r. In other words, ℓ can force a PEC with c. Assuming that c can sequentially propose a merger to both parties does not change the parameter space for which parties form mergers in equilibrium. However, in this case c would merge with ℓ for $\tilde{\psi} < \psi < \hat{\psi}$, and with r for $\psi < \tilde{\psi}$.

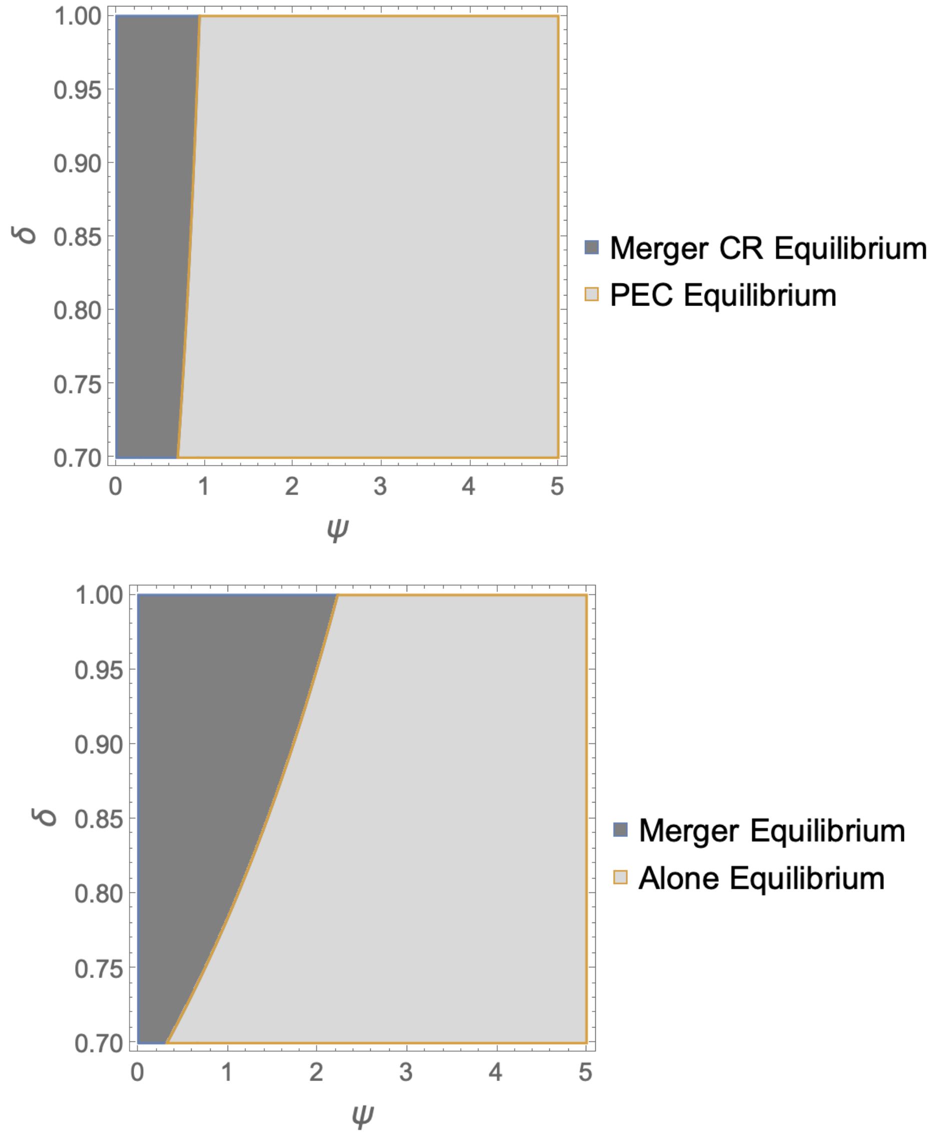

Figure 3 – Top Panel: $V_{c,1} < \max\{V_{\ell,1}, V_{r,1}\}$. Equilibrium featuring a merger between c and r (dark gray region) and a PEC between ℓ and c (light gray region) as a function of ψ (x axis) and parties' discount factor δ (y axis). Parameters are set to $z_\ell = -0.7$, $a = 1.5$, $\lambda = 0.5$. **Bottom Panel:** Equilibrium featuring merger (dark gray region) and parties running alone (light gray region) for the same parameter space.

Similarly, the bottom panel shows that mergers form in equilibrium for high volatility (dark gray region) even when c has a plurality of votes and could form a minority government thus enjoying all the rents from office by running alone. Conversely, when volatility is low c

prefers to run alone. Finally, note that the boundary between the two regions is not exactly vertical. A lower discount factor mutes the extent to which more electoral volatility results in a merger equilibrium. A binding alliance requires patience, because c could set its preferred policy in $t = 1$ but suffers a high policy cost in $t = 2$.

We can now go back to the initial example and interpret it through the lens of the model. One way to interpret the Dutch developments of the 1970s is as a movement from the low-volatility (light gray) area of the bottom panel to the high volatility (dark gray) area, which brought about the conditions for a merger among the three Dutch confessional parties. Similarly, the light gray region in the top panel can be thought of as the Italian political landscape before the 2006 election, when pre-electoral coalitions were common. The 2006 knife-edge election, which gave rise to volatility, could be thought as a movement towards the dark gray region of the graph, which incentivized parties across the political spectrum to merge.

5. Discussion

This section briefly discusses some of the model's assumptions and extensions.

While PECs allow parties to campaign autonomously, mergers demand that parties give up their ideological identities by forming new political entities. If voters are uncertain about the exact location of party platforms, they might evaluate differently a merger and a PEC between the same parties. An extension of the model incorporates voter uncertainty by introducing noise in the location of party platforms. To capture the fact that "mergers reduce, or even destroy, the information value of party labels for voters" (Ibenskas, 2016*a*, 343), I assume that mergers are associated with higher noise than PECs, and the noise is increasing in the distance between the constituent parties' bliss points. The analysis shows that mergers are not sustainable in equilibrium for high values of ideological uncertainty.

The model assumes that there are three competing parties. However, the same fundamental mechanism highlighted in the model would still hold with more than three parties, as long as parties outside the three in the model cannot find ways to make binding commitments. More parties would lead to more configurations of alliances for the centrist party to choose from, but electoral uncertainty and risk aversion would still create a room for mergers to emerge in equilibrium.

To see why, suppose that the centrist party faces an arbitrary number of parties on the left and the right, and that the probability that one extreme left party wins the election in the future is sufficiently high. In this case, the centrist party would merge to prevent the extreme party from winning. This form of "asymmetric" uncertainty captures well the reason behind the creation of the Japanese Liberal Democratic Party in 1955. While the Democrats were the biggest party at the time, the threat of a future Left government was decisive for the merger with the Liberals, which contributed to the stabilization of the Japanese governments in the decades that followed (Kohno, 1997).

6. Conclusion

Parties in multi-party systems often form alliances that have a significant impact on the development of party systems. Sometimes, these alliances are formed after elections. Other times, parties try to preempt post-electoral negotiations and form coalitions before elections, and sometimes they merge. To understand the incentives behind different types of alliances, this paper focuses on party response to the uncertainty introduced by anticipated shifts in voter support.

The central result of the model is that parties merge when there is high electoral volatility. The key intuition behind this result is that mergers are a form of insurance against potentially harmful outcome produced by significant shifts in voter support. Importantly, this result does not depend on whether the centrist party has a plurality of votes or not, and is robust to considering the possibility of (pre-electoral) bargaining and costly mergers. Recent political developments have brought attention to the electoral decline of established parties and the burst of electoral volatility in Europe. The model suggests that an increase in electoral volatility—possibly due to financial, health or war crises—might lead to an increase in the number of long-term coalitions in the future.

The important dynamic the model is intended to capture is the incentives confronting parties to form different types of alliances, in political systems with multiple parties running and playing an important role in formulating policy proposal. As such, the most natural application of this logic is given by parliamentary systems. Indeed, assuming that coalition members can implement platforms they bargain over most naturally reflects parliamentary systems without veto players who can alter the implemented policy. At the same time, the model's main insights could also be applied to presidential systems featuring multiple parties: In those systems as well parties propose policies, only under the additional constraints of a potential veto by the president.

This paper begins to unpack the incentives behind different forms of pre-electoral alliances. A promising area for future theoretical research could investigate, in addition to the motivations to coalesce into new parties, how intra-party factional incentives to split change as a function of electoral volatility and party organizational choices such as the decision to merge. Additionally, while the paper operates under the assumption that mergers are enduring, it would be interesting to characterize conditions under which, once they occur, mergers are stable and when we instead observe cycles of splits and mergers.

Another fruitful line of inquiry would be to examine how party formation can influence the likelihood of voter participation risks and party entry. The merging of parties might cause extremist followers of a particular constituent group to become disheartened and abstain from voting, potentially causing new parties to enter the political landscape. Further research could explore the extent to which the supporter base and ideological priorities of potential coalition partners factor into their choices to collaborate or go solo, and how these decisions ultimately impact voter turnout and the emergence of new party contenders.

In conclusion, the findings of this study suggest that electoral volatility through the insurance logic creates incentives to merge, which in turn increases electoral predictability. At the same time, it is possible that the very fact of parties merging has an endogenous effect on voters' preferences. It follows that political elites can use the available choices at hand to achieve greater stability and coherence in voter preferences, leading to more predictable electoral outcomes. This observation opens up promising possibilities for future research, specifically in exploring the potential interplay between party choices and electorate decision-making, and how these factors evolve and interact dynamically over time.

References

Austen-Smith, David and Jeffrey Banks. 1988. "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review* 82(2):405–422.

Bandyopadhyay, Siddhartha, Kalyan Chatterjee and Tomas Sjostrom. 2011. "Pre-electoral Coalitions and Post-Election Bargaining." *Quarterly Journal of Political Science* 6(1):1–53.

Baron, David P and John A Ferejohn. 1989. "Bargaining in legislatures." *American Political Science Review* 83(4):1181–1206.

Birch, Sarah. 2003. *Electoral Systems and Political Transformation in Post-Communist Europe*. Springer.

Blais, André and Indridi H Indridason. 2007. "Making Candidates Count: the Logic of Electoral Alliances in Two-Round Legislative Elections." *The Journal of Politics* 69(1):193–205.

Buisseret, Peter. 2017. "Electoral Competition with Entry under Non-Majoritarian Run-Off Rules." *Games and Economic Behavior* 104:494–506.

Buisseret, Peter and Richard Van Weelden. 2020. "Crashing the Party? Elites, Outsiders, and Elections." *American Journal of Political Science* 64(2):356–370.

Carroll, Royce and Gary W Cox. 2007. "The logic of Gamson's Law: Pre-election Coalitions and Portfolio Allocations." *American Journal of Political Science* 51(2):300–313.

Clark, William Roberts and Matt Golder. 2006. "Rehabilitating Duverger's theory: Testing the Mechanical and Strategic modifying Effects of Electoral Laws." *Comparative Political Studies* 39(6):679–708.

Coppedge, Michael. 1998. "The Dynamic Diversity of Latin American Party Systems." *Party Politics* 4(4):547–568.

Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge University Press.

Dalton, Russell J and Scott E Flanagan. 2017. *Electoral Change in Advanced Industrial Democracies: Realignment or Dealignment?* Vol. 5045 Princeton University Press.

Dhillon, Amrita. 2005. "Political Parties and Coalition Formation." *G Demange, G and M. Wooders, M.(Eds), Group Formation in Economics: Networks, Clubs and Coalitions, Cambridge University Press, New York* pp. 289–311.

Franklin, Mark N and Thomas T Mackie. 1983. "Familiarity and Inertia in the Formation of Governing Coalitions in Parliamentary Democracies." *British Journal of Political Science* 13(3):275–298.

Frey, Anderson, Gabriel López-Moctezuma and Sergio Montero. 2021. "Sleeping with the Enemy: Effective Representation under Dynamic Electoral Competition." *American Journal of Political Science* .

Gladdish, Ken. 1991. *Governing from the Centre: Politics and Policy-making in the Netherlands*. Hurst & Company.

Golder, Sona N. 2005. "Pre-Electoral Coalitions in Comparative Perspective: A test of Existing Hypotheses." *Electoral studies* 24(4):643–663.

Golder, Sona Nadenichek. 2006*a*. *The Logic of Pre-Electoral Coalition Formation*. Ohio State University Press.

Golder, Sona Nadenichek. 2006*b*. "Pre-Electoral Coalition Formation in Parliamentary Democracies." *British journal of political science* 36(2):193–212.

Ibenskas, Raimondas. 2016*a*. "Marriages of Convenience: Explaining Party Mergers in Europe." *The Journal of Politics* 78(2):343–356.

Ibenskas, Raimondas. 2016*b*. "Understanding Pre-Electoral Coalitions in Central and Eastern Europe." *British Journal of Political Science* 46(4):743–761.

Ibenskas, Raimondas and Allan Sikk. 2017. "Patterns of Party Change in Central and Eastern Europe, 1990–2015." *Party Politics* 23(1):43–54.

Kaminski, Marek M. 2001. "Coalitional Stability of Multi-Party Systems: Evidence from Poland." *American Journal of Political Science* pp. 294–312.

Kitschelt, Herbert, Zdenka Mansfeldova, Radoslaw Markowski and Gabor Toka. 1999. *Post-Communist Party Systems: Competition, Representation, and Inter-Party Cooperation*. Cambridge University Press.

Kohno, Masaru. 1997. *Japan's Postwar Party Politics*. Princeton University Press.

Koole, Ruud. 1994. "The Vulnerability of the Modern Cadre Party in the Netherlands." *How Parties Organize: Change and Adaptation in Party Organizations in Western Democracies* pp. 278–303.

Kuenzi, Michelle and Gina Lambright. 2001. "Party System Institutionalization in 30 African Countries." *Party Politics* 7(4):437–468.

Laver, Michael and Kenneth Benoit. 2015. "The Basic Arithmetic of Legislative Decisions." *American Journal of Political Science* 59(2):275–291.

Levy, Gilat. 2004. "A Model of Political Parties." *Journal of Economic theory* 115(2):250–277.

Lijphart, Arend. 1989. "From the Politics of Accommodation to Adversarial Politics in the Netherlands: A Reassessment." *West European Politics* 12(1):139–153.

Martin, Lanny W and Randolph T Stevenson. 2010. "The Conditional Impact of Incumbency on Government Formation." *American Political Science Review* pp. 503–518.

Morelli, Massimo. 2004. "Party Formation and Policy Outcomes under Different Electoral Systems." *The Review of Economic Studies* 71(3):829–853.

Pedersen, Mogens N. 1979. "The Dynamics of European Party Systems: Changing Patterns of Electoral Volatility." *European Journal of Political Research* 7(1):1–26.

Romer, Thomas and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice* pp. 27–43.

Shamir, Michal. 1984. "Are Western Party Systems "Frozen"? A Comparative Dynamic Analysis." *Comparative Political Studies* 17(1):35–79.

Shin, Euncheol. 2019. "A Model of Pre-Electoral Coalition Formation." *Games and Economic Behavior* 118:463–485.

Snyder, James M and Michael M Ting. 2002. "An Informational Rationale for Political Parties." *American Journal of Political Science* pp. 90–110.

Strom, Kaare, Ian Budge and Michael J Laver. 1994. "Constraints on Cabinet Formation in Parliamentary Democracies." *American Journal of Political Science* pp. 303–335.

Taagepera, Rein and Bernard Grofman. 2003. "Mapping the Indices of Seats–Votes Disproportionality and Inter-Election Volatility." *Party Politics* 9(6):659–677.

Tavits, Margit. 2008. "On the Linkage Between Electoral Volatility and Party System Instability in Central and Eastern Europe." *European Journal of Political Research* 47(5):537–555.

Van Mierlo, Hans JGA. 1986. "Depillarisation and the Decline of Consociationalism in the Netherlands: 1970–85." *West European Politics* 9(1):97–119.

Supplementary Appendix to ***Electoral Volatility and Pre-Electoral Alliances***

Table of Contents

Appendix A: Proofs of Main Results

Proof of Proposition 1. To compute party i's vote share from running alone ($V_{i,2}$) it suffices to identify the location of the voter who is indifferent between each pair of parties. Let $v_{\ell c,2}$ denote the ideal point of the voter who is indifferent between ℓ and c in $t = 2$, where $v_{\ell c,2}$ is located at $(z_\ell + z_c)/2$. The voter who is indifferent between c and r, denoted by $v_{cr,2}$, is defined analogously. Then, the vote share of ℓ is the CDF of the distribution of voters' ideal points evaluated at $v_{\ell c,2}$. Let $\xi \in [-a, a]$. Since voters' bliss points are uniformly distributed on $\mathcal{Z}$, ℓ's vote share is simply:

$$V_{\ell,2} = \frac{2a + z_\ell - 2\xi}{4a},$$

which depends on the realization of the shock to voters' preferences. A positive (negative) realization of the shock shifts voters' ideal policies to the right (left) thereby increasing the vote share of party r (ℓ) by $|\xi|$. Similarly, $V_{c,2} = (1 - z_\ell)/4a = V_{c,1}$ and[17]

$$V_{r,2} = 1 - V_{\ell,2} - V_{c,2} = \frac{1}{2} - \frac{1 - 2\xi}{4a}. \tag{A-2}$$

The vote share of a PEC formed in the second period is derived analogously. Let $V^{\text{pec}}_{\ell c,2}$ be the vote share of a PEC between ℓ and c in $t = 2$. Similarly to $V_{\ell,2}$ (A-1), the PEC's vote share is computed by finding the location of the voter who is indifferent between $z^{\text{pec}}_{\ell c}$ and z_r. That is,

[17]Clearly, if $\xi > a$:

$$V_{c,2} = \frac{\frac{z_c + z_r}{2} + (a - \xi)}{2a}, \tag{A-1}$$

and $V_{r,2} = 1 - V_{c,2}$ (and analogously when $\xi < -a$).

$V^{\text{pec}}_{\ell c,2}$ solves

$$\frac{1}{2} + \frac{z^{\text{pec}}_{\ell c} + z_r - 2\xi}{4a} - V^{\text{pec}}_{\ell c,2} = 0, \tag{A-3}$$

which produces

$$V^{\text{pec}}_{\ell c,2} = \frac{2a - 2\xi + \lambda z_\ell + 1}{4a}. \tag{A-4}$$

Similarly, the vote share of a PEC between c and r is

$$V^{\text{pec}}_{cr,2} = \frac{2a - \lambda + 2\xi - z_\ell}{4a}. \tag{A-5}$$

Finally, recall that $z^m_{\ell c,2} = z^{\text{pec}}_{\ell c,2}$, which implies that the vote share of a merger formed in $t = 2$ is analogous to that of a PEC: i.e., $V^m_{\ell c,2} = V^{\text{pec}}_{\ell c,2}$ and $V^m_{cr,2} = V^{\text{pec}}_{cr,2}$.

In what follows I define threshold values of the shock realization that determine parties' equilibrium behavior in the second period.

Definition 1. *Let $\underline{\xi}(z_\ell)$ be the value of ξ such that ℓ's vote share $V_{\ell,2} > 1/2$ for $\xi < \underline{\xi}(z_\ell)$. It follows from the expression of $V_{\ell,2}$ (A-1) that $\underline{\xi} = \frac{z_\ell}{2}$.*

Similarly, let $\overline{\xi}(z_\ell)$ be the value of the shock realization such that r's vote share $V_{r,2} > 1/2$ for $\xi > \overline{\xi}(z_\ell)$. It follows from the expression of $V_{r,2}$ (A-2) that $\overline{\xi} = \frac{z_r}{2}$.

Let us first consider parties' decision when $\xi > \overline{\xi}$. When r has the majority of votes, by running alone, r can implement its preferred policy. Similarly, when $\xi < \underline{\xi}$ party ℓ runs alone and wins, hence the implemented policy is $\hat{x}_2 = z_\ell$. Hence, for $\xi < \underline{\xi}$ ($\xi > \overline{\xi}$) ℓ (r) rejects a PEC proposal from c and in equilibrium parties run alone in the second period.

When $\underline{\xi} < \xi < \overline{\xi}$, no party obtains an absolute majority *if all parties run alone*, yet a party that runs alone against a PEC could obtain a majority of votes. In particular, when parties form PECs, it could be that (i) $V^{\text{pec}}_{\ell c,2} > 1/2$, (ii) $V^{\text{pec}}_{cr,2} > 1/2$, or both. The following definition derives values of the shock realization that define each of these occurrences.

Definition 2. *Let $\underline{\xi}^{pec}(z_\ell)$ be the value of ξ such that $V^{pec}_{cr,2} > 1/2$ for $\xi > \underline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{cr,2}$ (A-5) that*

$$\underline{\xi}^{pec} = \frac{z_\ell + \lambda}{2}. \tag{A-6}$$

Similarly, let $\overline{\xi}^{pec}(z_\ell)$ be the value of ξ such that ℓ's vote share $V^{pec}_{\ell c,2} > 1/2$ for $\xi < \overline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{\ell c,2}$ (B-4) that

$$\overline{\xi}^{pec} = \frac{1 + \lambda z_\ell}{2} \tag{A-7}$$

Let us analyze c's decision when $\boxed{\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}}$. Definition 2 implies that for these values of the shock realization both PECs would reach an absolute majority if formed. Then, c's proposal determines which PEC is formed in equilibrium. Under the assumptions, both ℓ and r accept c's proposal—as running alone entails an expected policy loss—and in $t = 2$ a PEC is formed. Then, c's decision determines whether the PEC is between ℓ and c, between c and r, or whether parties run alone.

Definition 3. *Let $\underline{\xi}^{pl}(z_\ell)$ and $\overline{\xi}^{pl}(z_\ell)$ be the values of ξ such that $V_{c,2} > \max\{V_{r,2}, V_{\ell,2}\}$ for $\underline{\xi}^{pl} < \xi < \overline{\xi}^{pl}$. It follows from the expression of $V_{\ell,2}, V_{r,2}$ that*

$$\underline{\xi}^{pl}(z_\ell) = a + z_\ell - \frac{1}{2}, \tag{A-8}$$

and

$$\overline{\xi}^{pl}(z_\ell) = 1 - a - \frac{z_\ell}{2}. \tag{A-9}$$

Let $V_{c,2} > \max\{V_{r,2}, V_{\ell,2}\}$, which happens when $\underline{\xi}^{\text{pl}} < \xi < \overline{\xi}^{\text{pl}}$. In this case, party c compares the payoff from running alone to that of forming a PEC with ℓ. Formally, c runs alone if and only if:

$$R > 2(p-1)\lambda^2 \left(z_\ell^2 - 1\right). \tag{A-10}$$

Intuitively, when rents from office are high, c takes the policy gamble and enjoys full rents from office.

Let $V_{c,2} < \max\{V_{r,2}, V_{\ell,2}\}$. In this case, R needs to be shared whether c forms a PEC or not. The expected payoff from the post-electoral policy (unknown) is lower than the payoff from $z_{\ell c}^{\text{pec}}$, for any $\alpha \in (0,1)$. Thus, c does not run alone. Party c compares the payoff from forming a PEC with ℓ with that of a PEC with r: since z_ℓ is closer to c's bliss point and $\lambda > 0$, c strictly prefers to propose an alliance to ℓ.

Finally, it could be that only one PEC has the absolute majority of votes in the second period. Suppose that $\boxed{V_{\ell c,2}^{\text{pec}} > 1/2 > V_{cr,2}^{\text{pec}}}$. This implies that $V_{\ell,2} > \max\{V_{c,2}, V_{r,2}\}$ for most parameter values. More precisely, assuming that λ is not too large is enough to rule out the case $V_{r,2} > \max\{V_{c,2}, V_{\ell,2}\}$. In this case, if c were to propose a PEC to ℓ, ℓ would accept because it would be part of the governing coalition with certainty. In equilibrium, c proposes a PEC to ℓ because the expected payoff from the post-electoral policy (unknown) is lower that the payoff from $z_{\ell c}^{\text{pec}}$. Thus, for $\underline{\xi} < \xi < \underline{\xi}^{\text{pec}}$ a PEC between ℓ and c forms. If instead $V_{c,2} > \max\{V_{\ell,2}, V_{r,2}\}$, c forms a PEC with ℓ when Equation A-10 does not hold (i.e., as long as rents from office are not too high), and runs alone otherwise.

Finally, suppose that $\boxed{V^{\text{pec}}_{cr,2} > 1/2 > V^{\text{pec}}_{\ell c,2}}$, which implies that $V_{r,2} > \max\{V_{\ell,2}, V_{c,2}\}$. In this case, c cannot form a PEC with ℓ because the coalition would lose against r. Since c prefers the lottery between the two coalitions to a PEC with r, in equilibrium parties run alone for $\overline{\xi}^{\text{pec}} < \xi < \overline{\xi}$. $\square$

Proof of Proposition 2. Denote by $U_{i,2}(\neg m_1)$ the expected second-period payoff of party i, *when no merger formed in the first period*. Proposition 1 allows us to express $U_{i,2}(\neg m_1)$ as a function of electoral volatility. By the uniform assumption of the shock, the probability of ξ falling below some threshold x is $\Pr\{\xi < x\} = \frac{1}{2} + \frac{\psi}{2}(x)$, hence the expected payoff from the second period is:

$$\begin{aligned}U_{i,2}(\neg m_1) = &\left[\frac{1}{2} + \frac{\psi}{2}\left(\frac{z_\ell}{2}\right)\right] u_i(z_\ell) + \frac{1}{4}\psi(2a + z_\ell - 1)\left[u_i(z^{\text{pec}}_{\ell c}) + \frac{R}{2}\right] \\ &- \frac{1}{4}\psi(4a + 3z_\ell - 3)\left[\alpha u_i\left(z^{\text{pec}}_{\ell c}\right) + (1-\alpha)u_i\left(z^{\text{pec}}_{cr}\right) + R\right] \\ &+ \frac{1}{4}\psi(2a + \lambda z_\ell + z_\ell - 1)\left[u_i(z^{\text{pec}}_{\ell c}) + \frac{R}{2}\right] \\ &- \frac{\lambda z_\ell \psi}{4}\left[\alpha u_i\left(z^{\text{pec}}_{\ell c}\right) + (1-\alpha)u_i\left(z^{\text{pec}}_{cr}\right) + \frac{R}{2}\right] + \left(\frac{1}{2} - \frac{\psi}{4}\right) u_i(z_r). \qquad \text{(A-11)}\end{aligned}$$

When c has a plurality of votes, c compares the expected payoff from running alone:

$$U^{\text{al}}_c = R + \alpha u_c\left(z^{\text{pec}}_{\ell c}\right) + (1-\alpha)u_c\left(z^{\text{pec}}_{cr}\right) + \delta U_{i,2}(\neg m_1), \qquad \text{(A-12)}$$

to the expected payoff from merging with ℓ or r, which is derived next.

Let us first analyze what happens in the second period following a merger between ℓ and c in $t = 1$. Since the merger persists in $t = 2$, the analysis is straightforward. Let $\tilde{\xi}_\ell$ be the value of the shock realization such that a merger between ℓ and c obtains half of the vote share.

Given the assumptions:

$$\tilde{\xi}_\ell = \frac{(1 + z^m_{\ell c})}{2}.$$

Then, for $\xi < \tilde{\xi}_\ell$, the policy outcome is $\hat{x}_2 = z^m_{\ell c}$, otherwise it is $\hat{x}_2 = 1$. Similarly, suppose that a merger between c and r formed in $t = 1$. Let $\tilde{\xi}_r$ be the value of the shock realization such that a merger between c and r obtains half of the vote share, where $\tilde{\xi}_r = (z_\ell + z^m_{cr})/2$. For $\xi > \tilde{\xi}_r$, the policy outcome is $\hat{x}_2 = z^m_{cr}$, otherwise it is $\hat{x}_2 = z_\ell$.

Denote by $U_{i,2}(m_{\ell c,1})$ the expected second-period payoff of party i, *when a merger between ℓ and c formed in the first period*. We can express $U_{c,2}(m_{\ell c})$ as

$$U_{c,2}(m_{\ell c}) = \left[\frac{1}{2} + \frac{\psi}{4}\big(1 + z^m_{\ell c}\big)\right]\left(u_i(z^m_{\ell c}) + \frac{R}{2}\right) + \left[\frac{1}{2} - \frac{\psi}{4}\big(z^m_{\ell c} + 1\big)\right] u_i(1). \tag{A-13}$$

Similarly, the expected payoff of party c from a merger between c and r can be written as

$$U_{c,2}(m_{cr}) = \left[\frac{1}{2} - \frac{\psi}{4}\big(z_\ell + z^m_{cr}\big)\right]\left(u_c(z^m_{cr}) + \frac{R}{2}\right) + \left[\frac{1}{2} + \frac{\psi}{4}\big(z_\ell + z^m_{cr}\big)\right] u_c(z_\ell). \tag{A-14}$$

Given the expressions (A-13-A-14), we can easily compare party c's expected payoff from merging with ℓ and r. The expected payoff of party c from a merger between ℓ and c is

$$U^m_{c,\ell c} = u_c(z^m_{\ell c}) + \frac{R}{2} + \delta U_{c,2}(m_{\ell c}), \tag{A-15}$$

where the realized policy in the first period coincides with the merged party's platform, since the merger has the majority of votes in $t = 1$. The expression for $U^m_{i,cr}$ is analogous.

Differentiating $U_c^{\mathrm{al}}(\psi) - U_{c,\ell c}^m$ with respect to ψ yields:

$$\delta(-2R(a + z_\ell - 1) + \lambda^2(z_\ell - 1)(4a(\alpha - 1)(z_\ell + 1) - 3\alpha + (3\alpha - 2)z_\ell^2 + 3)$$
$$- z_\ell^3 + (\alpha - 1)\lambda^3(z_\ell^2 - 1)z_\ell + 1) - \frac{1}{8}az_\ell(R - 2(\lambda - \psi - 1)(\lambda + 3\psi - 1)) + a\psi,$$

which is always positive as long as R is low enough: as volatility goes down (ψ goes up), the payoff from running alone increases.

Let $\hat{\psi}^a$ be the value of volatility such that $U_c^{\mathrm{al}}(\psi) - U_{c,\ell c}^m(\psi) = 0$. It is easy to show that a real root that solves $U_c^{\mathrm{al}}(\psi) = U_{c,\ell c}^m(\psi)$ exists (the expression is lengthy and therefore omitted). It follows from the previous step of the proof that c runs alone for $\psi > \hat{\psi}^a$ and prefers to form a merger with the closest party ℓ for $\psi < \hat{\psi}^a$.

Let $\Delta_i^m(\psi) = U_{i,\ell c}^m - U_{i,cr}^m$. Differentiating $\Delta_c^{\mathrm{m}}(\psi)$ with respect to ψ yields

$$\frac{\partial \Delta_c^{\mathrm{m}}(\psi)}{\partial \psi} = \frac{\delta(1 + z_\ell)}{8}\left((\lambda + 1)R - 2(\lambda - 1)\left(\lambda^2 + \lambda + \left(\lambda^2 + \lambda + 1\right)^2 - \left(\lambda^2 + 1\right) + 1\right)\right), \quad \text{(A-16)}$$

which is always negative.

Let $\tilde{\psi}$ be the value of ψ such that $U_{c,\ell c}^m = U_{c,cr}^m$. Solving for ψ produces:

$$\tilde{\psi} = \frac{4(z_\ell - 1)\left((\delta + 2)\lambda^2 - \delta\right)}{\delta\left(-2\lambda^3 + (\lambda + 1)R - 2\left(\lambda^3 - 1\right)z_\ell^2 + 2(\lambda - 1)\left(\lambda^2 + 1\right)z_\ell + 2\right)}. \quad \text{(A-17)}$$

It follows from the previous step of the proof that $U_{c,cr}^m > U_{c,\ell c}^m$ for $\psi < \tilde{\psi}$. □

Proof of Remark 1. Differentiating $\Delta_c^{\mathrm{m}}/\psi$ with respect to z_ℓ yields:

$$-\frac{1}{4}\delta(\lambda - 1)\left(\lambda + 3\left(\lambda^2 + \lambda + 1\right)z_\ell^2 + 2\lambda z_\ell\right), \quad \text{(A-18)}$$

which is always positive. ☐

Proof of Proposition 3. Differentiating $U_c^{\text{al}} - U_{c,\ell c}^m$ with respect to R yields

$$\frac{\partial\left(U_c^{\text{al}} - U_{c,\ell c}^m\right)}{\partial R} = \frac{1}{2} + \frac{(z_\ell(\lambda - \psi - 1) - 4\delta\psi - 2) - 4\delta\psi(z_\ell - 1)}{8}, \tag{A-19}$$

which is always positive. ☐

Proof of Lemma 1. The expected payoff of party i from a PEC between ℓ and c is:

$$U_{i,\ell c}^{\text{pec}} = u_i(z_{\ell c}^{\text{pec}}) + \frac{R}{2} + \delta U_{i,2}(\neg m_1), \tag{A-20}$$

where the last component of the RHS is party i's expected payoff in $t = 2$ when no merger is formed in $t = 1$ (A-11). The expressions for $U_{i,cr}^{\text{pec}}$ is analogous, substituting $u_i(z_{cr}^{\text{pec}})$ into the first-period payoff.

The difference $U_{c,\ell c}^m - U_{c,\ell c}^{\text{pec}}$ simplifies to:

$$\frac{\delta}{8}\left[R((\lambda+1)\psi z_\ell + 2) + 2z_\ell\left(\psi\left((\alpha-1)\lambda^3 + \lambda - z_\ell^2\left(\alpha\lambda^3 + \lambda^2 - 1\right)\right) - 2\left(\lambda^2 - 1\right)z_\ell\right)\right]. \tag{A-21}$$

Differentiating (A-21) with respect to ψ produces:

$$\frac{\delta z_\ell}{8}\left(2\left((\alpha-1)\lambda^3 + \lambda\right) + (\lambda+1)R - 2z_\ell^2\left(\alpha\lambda^3 + \lambda^2 - 1\right)\right), \tag{A-22}$$

which is always negative.

Let $\hat{\psi}_c$ be the value of ψ such that $U^{m}_{c,\ell c} = U^{\text{pec}}_{c,\ell c}$, where

$$\hat{\psi}_c = \frac{2\left[R - 2\left(\lambda^2 - 1\right) z_\ell^2\right]}{z_\ell\left[-2\left((\alpha-1)\lambda^3 + \lambda\right) - (\lambda+1)R + 2z_\ell^2\left(\alpha\lambda^3 + \lambda^2 - 1\right)\right]} \tag{A-23}$$

It follows from the first step of the proof that for $\psi > \hat{\psi}_c$, $U^{\text{pec}}_{c,\ell c} > U^{m}_{c,\ell c}$. □

Proof of Proposition 4. For c to prefer a merger with ℓ, it must be that (i) $U^{m}_{c,\ell c} > U^{m}_{c,cr}$, (ii) $U^{m}_{c,\ell c} > U^{\text{pec}}_{c,\ell c}$, (iii) $U^{m}_{c,\ell c} > U^{\text{pec}}_{c,cr}$ and (iv) $U^{m}_{c,\ell c} > U^{\text{al}}_{c}$. Note that we can immediately compare the expected payoff from the two PECs, because the second period payoff is the same for both of them (A-11). This leads to the following strict ranking for party c: $U^{\text{pec}}_{c,\ell c} > U^{\text{pec}}_{c,cr}$, which simply follows from comparing the first-period payoffs. Clearly, conditions (i)-(iv) are necessary but not sufficient for a merger between c and ℓ to form in equilibrium, as the merger must be incentive compatible for ℓ as well. It is straightforward to derive similar rankings for ℓ and r respectively: $U^{\text{pec}}_{\ell,\ell c} > U^{\text{pec}}_{\ell,cr}$ and $U^{\text{pec}}_{r,cr} > U^{\text{pec}}_{r,\ell c}$.

Consider the following equilibrium: c proposes a PEC to ℓ for $\psi > \hat{\psi}_c$, and a merger to ℓ for $\psi < \hat{\psi}_c$, where $\hat{\psi}_c$ is defined in (A-23). Party ℓ accepts a PEC proposal for $\psi > \hat{\psi}_c$ because it prefers a PEC with c to the alternatives from rejection: if ℓ rejects c's offer, c proposes a PEC to r, which always accepts. Recall that no party has an absolute majority of votes, which implies that the PEC with policy z^{pec}_{cr} would win against party ℓ. Since $z^{\text{pec}}_{\ell c} \succ_\ell z^{\text{pec}}_{cr}$ for all $\lambda > 0$ and $R > 0$, ℓ accepts c's proposal for $\psi > \hat{\psi}_c$, and in equilibrium a PEC between c and ℓ forms in $t = 1$.

For $\psi < \hat{\psi}_c$, c prefers to form a merger rather than a PEC with ℓ. Suppose that for $\psi < \hat{\psi}_c$ c proposes a merger to ℓ. To show that ℓ accepts, it suffices to compare ℓ's continuation value from merging with c to that of a PEC with c (i.e., what c would propose following a rejection)—

since the first-period payoff is identical in both cases. Denote by $\hat{\psi}_\ell$ the value of ψ that makes ℓ indifferent between merging and forming a PEC with c, where:

$$\hat{\psi}_\ell = \frac{2\left(R + 2(\lambda-1)^2 z_\ell^2\right)}{z_\ell\left(2\left((\alpha-1)\lambda^3+\lambda\right)+\alpha\lambda R - R - 2z_\ell^2\left(\alpha\lambda^3+(1-2\alpha)\lambda^2-2\lambda+1\right)-4\lambda z_\ell((\alpha-1)\lambda+1)\right)}, \tag{A-24}$$

Comparing the expression of $\hat{\psi}_\ell$ with (A-23), we see that $\hat{\psi}_c < \hat{\psi}_\ell$ as long as λ is not too high. Intuitively, because of concave preferences, when volatility is high ($\psi < \hat{\psi}_c$) ℓ suffers more than c from the event of r winning a majority in $t = 2$. It follows that ℓ accepts c's proposal and a merger between ℓ and c forms for $\psi < \hat{\psi}_c$.

We are left to check whether a merger between c and r can form for some ψ. From the proof of Proposition 2, we have that $U^m_{c,cr} > U^m_{c,\ell c}$ for $\psi < \tilde{\psi}$. Suppose that, for $\psi < \tilde{\psi}$, c proposes a merger to the extreme party r. Rejecting is strictly dominated for r, since the difference $U^m_{r,cr} - U^{\text{pec}}_{r,\ell c}$, which can be expressed as

$$-\frac{1}{8}R(\delta(\psi(\lambda+(\alpha-1)\lambda z_\ell+z_\ell-2)+2)-4)-\frac{1}{4}\delta(-\psi+\lambda^3\psi\left((\alpha-1)z_\ell^3-\alpha z_\ell+z_\ell-1\right)$$
$$-\lambda\left(\psi z_\ell^2-2\psi z_\ell+4\right)+\lambda^2\left(\psi\left(z_\ell^3-2\alpha z_\ell^2+z_\ell^2+2\alpha z_\ell-3z_\ell+2\right)+2\right)+2)+\lambda(z_\ell-1)(\lambda+\lambda z_\ell-2),$$

is positive under the assumptions. Hence, r always accepts a merger proposal from c. From the previous step of the proof it follows that in equilibrium a merger between c and r forms for $\psi < \tilde{\psi}$ and a PEC between c and ℓ forms for $\psi > \tilde{\psi}$. □

Appendix B: Coalition Platforms as a function of Vote Shares

Denote by $V_{i,t}$ party i's vote share at time $t = \{1, 2\}$, where $V_{i,1} < 1/2$ for each party i. Suppose that ℓ and c merge or form a PEC in t. Then, the policy platform of the resulting party or PEC in t is a convex combination of the constituent parties' bliss points:

$$z^{m}_{\ell c,t} = z^{\text{pec}}_{\ell c,t} = \lambda_{\ell,t}\, z_\ell + (1 - \lambda_{\ell,t})\, z_c. \tag{B-1}$$

The weight $\lambda_{\ell,t} \in (0, 1)$ measures the relative electoral strength of the extreme party (ℓ) in t, which depends on the parties' vote shares as follows:

$$\lambda_{\ell,t} = \frac{1}{2} + \phi(V_{\ell,t} - V_{c,t}), \tag{B-2}$$

where the parameter $\phi \in \mathbb{R}_+$ is a normalization ensuring that $\lambda_{\ell,t} \in (0, 1)$. Equation 1 implies that the policies resulting from PECs and mergers are equivalent *only in the same period*. Because of the electoral shock, the policy resulting from a merger (or PEC) formed in $t = 2$ is different from the policy resulting from a merger formed in $t = 1$ and persisting in $t = 2$. This is because the shock changes parties' relative vote shares and in turn the weight each party has in the common platform. Crucially, while mergers "solidify" the relative power parties have in $t = 1$—which is given by each party's vote share $V_{i,1}$—PECs are re-negotiated in $t = 2$, allowing parties to be flexible to changes in the electoral environment.

The analysis largely proceeds as in the baseline model. In the remainder of the section, I will highlight the differences that emerge from having platforms dependent on vote shares. In this and the following extensions I will make the following assumptions that do not change the main qualitative results but help simplifying the expressions considerably. First, I remove from parties' payoff the additional motive of rents from office and simply focus on the policy payoff component. Second, I assume that the implemented policy after the election is the one preferred by whoever has a plurality of votes (instead of being unknown and a function of α).

I begin with the second period. Let $V^{\text{pec}}_{lc,2}$ be the vote share of a PEC between ℓ and c in $t = 2$. Similarly to $V_{\ell,2}$ (A-1), the PEC's vote share is computed by finding the location of the voter who is indifferent between $z^{\text{pec}}_{lc,2} = \lambda_{\ell,2}\, z_\ell + (1 - \lambda_{\ell,2})\, z_c$ and $z_{r,2}$. That is, $V^{\text{pec}}_{lc,2}$ solves

$$\frac{1}{2} + \frac{z^{\text{pec}}_{lc,2}(V^{\text{pec}}_{lc,2}) + z_r - 2\xi}{4a} - V^{\text{pec}}_{lc,2} = 0, \tag{B-3}$$

which produces

$$V^{\text{pec}}_{lc,2} = \frac{8a^2 + 2a(-4\xi + z_\ell\phi + z_\ell + 2) + z_\ell\phi(-2\xi + 2z_\ell - 1)}{16a^2}. \tag{B-4}$$

Similarly, the vote share of a PEC between c and r is

$$V^{\text{pec}}_{cr,2} = \frac{8a^2 - 2a(-4\xi + 2z_\ell + \phi + 1) - \phi(2\xi + z_\ell - 2)}{16a^2}. \tag{B-5}$$

Recall that $z^m_{lc,2} = z^{\text{pec}}_{lc,2}$ (1), which implies $V^m_{lc,2} = V^{\text{pec}}_{lc,2}$ and $V^m_{cr,2} = V^{\text{pec}}_{cr,2}$. What determines parties' choice in the second period? The shock has a twofold impact on parties' decision: first, it has a *direct* effect on parties' vote share, by swinging voters' preferences in favor of either ℓ or r. I denote this the *electoral effect*. Second, by changing parties' relative vote share, the shock *indirectly* affects parties' influence on the final policy of a PEC. I denote this the *policy effect*. In what follows I define threshold values of the shock realization that determine which of these two effects prevails in parties' decision to form a PEC in $t = 2$. These values also provide useful cutoffs to describe parties' equilibrium behavior in the second period.

Definition 4. *Let $\underline{\xi}(z_\ell)$ be the value of ξ such that ℓ's vote share $V_{\ell,2} > 1/2$ for $\xi < \underline{\xi}(z_\ell)$. It follows from the expression of $V_{l,2}$ (A-1) that $\underline{\xi} = \frac{z_l}{2}$.*

Similarly, let $\overline{\xi}(z_\ell)$ be the value of the shock realization such that r's vote share $V_{r,2} > 1/2$ for $\xi > \overline{\xi}(z_\ell)$. It follows from the expression of $V_{r,2}$ (A-2) that $\overline{\xi} = \frac{z_r}{2}$.

Let us first consider parties' decision when $\xi > \overline{\xi}$. When a party has the majority of votes, the electoral effect trumps every other consideration: by running alone, r can implement its preferred policy. Similarly, when $\xi < \underline{\xi}$ party ℓ runs alone and wins, hence the implemented policy is $\hat{x}_2 = z_\ell$. Hence, for $\xi < \underline{\xi}$ ($\xi > \overline{\xi}$) ℓ (r) rejects a PEC proposal from c and in equilibrium parties run alone in the second period.

When $\underline{\xi} < \xi < \overline{\xi}$, no party obtains an absolute majority *if all parties run alone*, yet a party that runs alone against a PEC could obtain a majority of votes. In particular, when parties form PECs, it could be that (i) $V^{\text{pec}}_{lc,2} > 1/2$, (ii) $V^{\text{pec}}_{cr,2} > 1/2$, or both. The following definition derives values of the shock realization that define each of these occurrences.

Definition 5. *Let $\underline{\xi}^{pec}(z_\ell)$ be the value of ξ such that $V^{pec}_{cr,2} > 1/2$ for $\xi > \underline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{cr,2}$ (B-5) that*

$$\underline{\xi}^{pec} = \frac{2a(2z_\ell + \phi + 1) + (z_\ell - 2)\phi}{8a - 2\phi}. \tag{B-6}$$

Similarly, let $\overline{\xi}^{pec}(z_\ell)$ be the value of ξ such that ℓ's vote share $V^{pec}_{lc,2} > 1/2$ for $\xi < \overline{\xi}^{pec}(z_\ell)$. It follows from the expression of $V^{pec}_{lc,2}$ (B-4) that

$$\overline{\xi}^{pec} = \frac{2a(z_\ell \phi + z_\ell + 2) + z_\ell(2z_\ell - 1)\phi}{8a + 2z_\ell \phi}. \tag{B-7}$$

Let us analyze c's decision when $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$. Definition 5 implies that for these values of the shock realization both PECs would reach an absolute majority. Then, c's proposal determines which PEC is formed in equilibrium. Under the assumptions, both ℓ and r accept c's proposal—as running alone would result in a certain loss—and in $t = 2$ a PEC is formed. Then, c's decision determines whether the PEC is between ℓ and c or between c and r.[18] c compares the payoff from forming a PEC with ℓ, i.e.,

$$u_c(z^{\text{pec}}_{lc,2}) = -\frac{z_\ell^2(2a(\phi+1) + \phi(2z_\ell - 2\xi - 1))^2}{16a^2}, \tag{B-8}$$

with the payoff from forming a PEC with r

$$u_c(z^{\text{pec}}_{cr,2}) = -\frac{(2a(\phi+1) + \phi(2\xi + z_\ell - 2))^2}{16a^2}. \tag{B-9}$$

The following results show how c's decision changes with different values of the shock realization and with the location of parties' platforms. In particular, Lemma B-1 shows that, as voters' preferences shift in favor of r (ℓ), the centrist party prefers a coalition with ℓ (r). Lemma B-2 then shows that c prefers an alliance with the ideologically closest party when

[18] Running alone is strictly dominated for c, because it would result in the adoption of the policy preferred by the party with the plurality of votes.

voters' preferences are stable (i.e., $\xi = 0$). Finally, Proposition B-1 characterizes the (second period) equilibrium alliance configuration based on the value of the shock realization.

Lemma B-1. Policy Effect. *Let* $\Delta_c^{pec}(\xi) = u_c(z_{lc,2}^{pec}) - u_c(z_{cr,2}^{pec})$. $\Delta_c^{pec}(\xi)$ *is strictly increasing in* ξ.

Proof. Let $\Delta_c^{\text{pec}}(\xi) = u_c(z_{lc,2}^{\text{pec}}) - u_c(z_{cr,2}^{\text{pec}})$, where

$$\Delta_c^{\text{pec}}(\xi) = \frac{(2a(\phi+1) + \phi(2\xi + z_\ell - 2))^2 - z_\ell^2(2a(\phi+1) + \phi(-2\xi + 2z_\ell - 1))^2}{16a^2}.$$

Differentiating Δ_c with respect to ξ yields

$$\frac{\partial \Delta_c}{\partial \xi} = \frac{\phi\left(2a\left(z_\ell^2+1\right)(\phi+1) + (z_\ell - 1)\phi\left(-2\xi + 2z_\ell^2 - 2\xi z_\ell + z_\ell + 2\right)\right)}{4a^2}$$

which is always positive. □

When $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$ both PECs obtain a majority if formed. When this is the case, Lemma B-1 shows that the policy effect determines c's proposal decision. To see why, suppose that the shock realization is such that c is indifferent between the two coalitions. Now, let the value of the shock realization increase. This increase leads to a higher (lower) vote share of party r (ℓ), which means that r (ℓ)'s preferred policy weighs more (less) in a PEC between c and r (ℓ). Then, ceteris paribus, c would prefer to form a PEC with ℓ. Conversely, a lower value of the shock makes a coalition with r more appealing.

Analogously to the Baron and Ferejohn (1989) model where the proposer chooses a coalition with the partner who has the lower recognition probability, this policy effect prevails whenever c could achieve a majority by forming a PEC with both parties (i.e., when $\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$).

Whether c forms a PEC with ℓ or r ultimately depends on the location of the platform z_ℓ. When $\xi = 0$, c is indifferent between ℓ and r (i.e., $\Delta_c^{\text{pec}}(0) = 0$) when z_ℓ and z_r are equidistant from z_c, and prefers the closer ally otherwise, as the next result shows.

Lemma B-2. $\Delta_c^{pec}(0)$ *is strictly increasing in* z_ℓ*.*

Proof. Differentiating Δ_c with respect to z_ℓ yields

$$\frac{\partial \Delta_c}{\partial z_\ell} = \frac{-2a^2 z_\ell(\phi+1)^2 - a\left(6z_\ell^2 - 2z_\ell - 1\right)(\phi+1)\phi + \left(-4z_\ell^3 + 3z_\ell^2 - 1\right)\phi^2}{4a^2},$$

which is always positive. □

Since $z_\ell \in (0,1)$, a corollary of Lemma B-2 is that when $\xi = 0$ party c prefers a coalition with ℓ. Furthermore, Lemma B-1 implies that when the shock favors r, c continues to prefer an alliance with ℓ. The next definition derives the value of the shock realization, $\hat{\xi}$, such that party c is indifferent between proposing a PEC to ℓ or r (i.e., $\Delta_c^{\text{pec}}(\hat{\xi}) = 0$) for any z_ℓ.

Definition 6. *Let* $\hat{\xi}(z_\ell)$ *be the value of the shock realization such that* $\Delta_c^{pec}(\hat{\xi}) = 0$*. It follows from the expression of* Δ_c^{pec} *(B-8-B-9) that*

$$\hat{\xi} = \frac{a(z_\ell - 1)(\phi+1) + \left(z_\ell^2 - z_\ell + 1\right)\phi}{(z_\ell+1)\phi}. \tag{B-10}$$

It follows from Lemma B-1 that c prefers to form a PEC with ℓ (r) when $\xi > \hat{\xi}$ ($\xi < \hat{\xi}$). Whenever both PECs obtain the majority of votes ($\underline{\xi}^{\text{pec}} < \xi < \overline{\xi}^{\text{pec}}$), the threshold $\hat{\xi}$ determines which of the two PECs form.

Finally, it could be that only one PEC has the absolute majority of votes in the second period. Suppose that $V^{\text{pec}}_{lc,2} > 1/2 > V^{\text{pec}}_{cr,2}$, which implies $V_{\ell,2} > \max\{V_{c,2}, V_{r,2}\}$.[19] If c were to propose a PEC to ℓ, ℓ would reject because it could set its preferred platform by forming a minority government after elections. Similarly, because ℓ has a relative majority, a PEC between c and r would not change the post-electoral policy set by ℓ. Hence, when only a PEC between ℓ and c reaches the absolute majority of votes, in equilibrium parties run alone and ℓ forms a minority government (the case such that $V^{\text{pec}}_{cr,2} > 1/2$ is analogous).

The following proposition summarizes the last observation and the previous results without proof, showing when parties form alliances or run alone in the second period, when no mergers form in the first period.

Proposition B-1. Second-Period Policy Outcome. *If $V_{c,2} > V_{\ell,2}, V_{r,2}$, in equilibrium parties run alone in $t = 2$. Suppose that c has no plurality, and that no merger formed in $t = 1$. Then in $t = 2$ parties form PECs for intermediate realizations of the shock ξ, and compete alone for extreme ones. In particular, for $\underline{\xi}^{pec} < \xi < \hat{\xi}$ ($\hat{\xi} < \xi < \overline{\xi}^{pec}$), a PEC between c, r (c, ℓ) forms, and $\hat{x}_2 = z^{pec}_{cr,2}$ ($z^{pec}_{lc,2}$). Conversely, when $\xi < \underline{\xi}^{pec}$ ($\xi > \overline{\xi}^{pec}$), parties run alone and $\hat{x}_2 = z_\ell$ (z_r).*

Compared to the baseline model, Proposition B-1 also shows that the policy outcome is non-monotonic in the rightist party's relative popularity: that is, it could be that the implemented policy shifts to the right as ℓ becomes more popular. This is a consequence of the pivotality of the centrist party in choosing coalitions: as the policy cost of an alliance with ℓ increases, ceteris paribus c prefers the weaker party r.

I now turn to the first period analysis. Let $V_{c,1} > \max\{V_{l,1}, V_{r,1}\}$. In $t = 1$, c compares the expected payoff from running alone to that of forming a merger. Proposing a PEC is clearly

[19] It follows from Definition 5 that this is the case for $\underline{\xi} < \xi < \underline{\xi}^{\text{pec}}$.

dominated because $u_c(z_c) > u_c(z_{lc,1}^{\text{pec}}), u_c(z_{cr,1}^{\text{pec}})$. Denote by $U_{i,2}(\neg m_1)$ the expected second-period payoff of party i, *when no merger formed in the first period*. The expected payoff from the second period is:

$$U_{i,2}(\neg\, m_1) = \left[\frac{1}{2} + \frac{\psi}{2}\left(\underline{\xi}^{\text{pec}}\right)\right] u_i(z_\ell) + \left[\frac{\psi}{2}(\hat{\xi}) - \frac{\psi}{2}\left(\underline{\xi}^{\text{pec}}\right)\right] V_{i,2}(z_{cr,2}^{\text{pec}}) + \left[\frac{\psi}{2}\left(\overline{\xi}^{\text{pec}}\right) - \frac{\psi}{2}\left(\hat{\xi}\right)\right] V_{i,2}(z_{lc,2}^{\text{pec}}) + \left[\frac{1}{2} - \frac{\psi}{2}(\overline{\xi}^{\text{pec}})\right] u_i(z_r), \tag{B-11}$$

where $V_{i,2}(z_{lc,2}^{\text{pec}})$ is the expected payoff of party i from the LC coalition platform, which depends on the realization of the shock:

$$V_{i,2}(z_{lc,2}^{\text{pec}}) = \int_{\hat{\xi}}^{\overline{\xi}^{\text{pec}}} u_i\left(z_{lc,2}^{\text{pec}}\right) \frac{1}{\overline{\xi}^{\text{pec}} - \hat{\xi}}\, d\xi, \tag{B-12}$$

and analogously for $V_{i,2}(z_{cr,2}^{\text{pec}})$. When c has a plurality of votes, it compares the expected payoff from running alone, i.e. $U_c^{\text{alone}} = u_c(z_c) + \delta U_{i,2}(\neg\, m_1)$, to the expected payoff from merging with ℓ or r, which is as in the baseline model. Differently from the second-period, all the first-period analysis is equivalent to the baseline model, but the expressions are significantly more cumbersome. While I omit the expressions for their length, it can be shown that all the main results are unchanged (all the expressions and detailed proofs for the extensions are available upon request).

Appendix C: Introducing Uncertainty over Platforms' Location

While each party is associated with a particular policy (its "brand"), z_i, parties typically feature heterogeneous preferences inside them. This heterogeneity is crucial, as the policy platform that is chosen by each party in a given election might differ from its policy brand (or, in other words, parties cannot fully pre-commit to policies). This section formalizes this idea by introducing noise in the location of parties' platforms.

Let $x_{i,t}$ be the policy platform that is selected by party i in a given election. This platform corresponds to the realization of the random variable $X_{i,t} = z_i + \varepsilon$, where $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. The smaller ε, the sharpest the message of the party (i.e., the most informative the party brand). We can interpret the support of X_i as follows. Parties typically gather multiple candidates who are proponents of different issues, some of which might be very far from the party brand. Depending on which of these candidates wins the election, the party platform could differ from the ex-ante party brand.

When ℓ and c merge, the resulting platform is a random variable centered at $z^m_{\ell c,1}$, the convex combination of the constituent parties' bliss points:

$$X^m_{\ell c,1} = z^m_{\ell c,1} + \varepsilon^m, \tag{C-1}$$

where $\varepsilon^m \sim \mathcal{N}(0, \sigma^2_m)$, and

$$\sigma^2_m = \sigma^2 + \frac{|z_\ell - z_c|}{\gamma}. \tag{C-2}$$

By creating a new political entity, mergers decrease the informativeness of the constituent parties' brands: for any distinct pair of platforms z_ℓ and z_c, $\sigma^2_m > \sigma^2$ for any $\gamma \in \mathbb{R}_+$. The noise that arises from a merger is increasing in the distance between its constituent parties' bliss points: since voters expect candidates to be drawn from anywhere between z_c and z_ℓ, the uncertainty cost increases with the distance among platforms.[20] Furthermore, the noise is decreasing in γ: as $\gamma \to \infty$, $\sigma_m^2 \to \sigma^2$. As such, γ could be interpreted as the amount of trust between the merger's partners.[21] The merged party's brand $z_{\ell c,1}^m$ is a convex combination of the constituent parties' bliss points, as in the previous section: $z_{\ell c,1}^m = \lambda_{\ell,1} z_\ell + (1-\lambda_{\ell,1}) z_c$, where $\lambda_{\ell,1} = \frac{1}{2} + \phi\,(V_{\ell,1} - V_{c,1})$.

Differently from mergers, PECs preserve the identity of different parties. Thus, when two parties form a PEC the noise term is the same as when parties run individually: $\varepsilon \sim \mathcal{N}(0, \sigma^2)$.

Because parties cannot pre-commit to policies, voters do not know the exact policy each party selects and suffer an uncertainty cost which is captured by the variance of X_i. Formally, voter v's expected payoff from party i's platform is

$$\begin{aligned} EU_v(X_i) &= \mathbb{E}\big[-\big(X_i - z_v\big)^2\big] \\ &= -\big(z_i - z_v\big)^2 - \sigma^2, \end{aligned} \tag{C-3}$$

where $z_i = \mathbb{E}[X_i]$ and $\sigma^2 = \mathrm{Var}[X_i]$.[22]

To compute each party's vote share when parties run alone, we need to identify the location of the indifferent voter for each pair of parties. Since σ^2 is constant across parties, we can

[20] This assumption is supported by empirical evidence showing that mergers are more likely to form between ideologically close parties (Ibenskas, 2016*a*), which suggests that parties anticipate the electoral cost of merging.

[21] When deciding to merge, a party faces the risk that the other partner would renege on the agreement by increasing its policy influence above the agreed at the time of the merger. While I leave it exogenous, it is reasonable to think γ to be positively correlated with the constituent parties' previous experience of governing together, which can reduce the uncertainty about partners' behavior (Franklin and Mackie, 1983; Martin and Stevenson, 2010).

[22] The second equality follows from $\mathrm{Var}[X_i] = \mathbb{E}[X_i^2] - \mathbb{E}[X_i]^2 = \sigma^2$, which allows to re-express $EU_v(X_i)$ as

$$\begin{aligned} EU_v(X_i) &= -\sigma^2 - \mathbb{E}[X_i]^2 + 2\mathbb{E}[X_i] z_v - z_v^2 \\ &= -\big(\mathbb{E}[X_i]^2 - 2\mathbb{E}[X_i] z_v + z_v^2\big) - \sigma^2. \end{aligned}$$

focus on the comparison between pairs of party brands (z_ℓ, z_c and z_c, z_r), as in the baseline model. The same holds when evaluating a PEC's vote share.

The analysis changes when computing the vote share of a merger. Denote by $v^m_{lc,r,2}$ the voter who is indifferent between party r and a merger between ℓ and c in the second period. That is, $v^m_{lc,r,2}$ solves:

$$-\left(v^m_{lc,r,2} - z^m_{\ell c,2}\right)^2 - \frac{|z_\ell - z_c|}{\gamma} + \left(v^m_{lc,r,2} - z_{r,2}\right)^2 = 0. \tag{C-4}$$

From the indifference condition (C-4) it is clear that parties sacrifice at least some of their vote share when deciding to merge (vis-à-vis forming a PEC). This is because—when z_ℓ and z_c differ—voters pay an uncertainty cost when voting for a merged party. Despite this cost from merging, the next result shows that the trade-off identified in Proposition 4 holds, as long as the uncertainty cost associated to the merger is not too high.

Proposition C-1. Equilibrium with Electoral Uncertainty. *When γ is high enough, in equilibrium parties form mergers when electoral volatility is sufficiently high (low ψ), and PECs for low electoral volatility (high ψ). When γ is low, in equilibrium c forms a PEC with the closest party (ℓ).*

Proof. The analysis of $t = 2$ is analogous to the baseline model. First, suppose that no merger formed in $t = 1$. Because $\sigma^2_m > \sigma^2$, mergers are dominated in the second period, and both voters' and parties' decision are identical to the baseline.

Suppose instead that a merger between C and R formed in $t = 1$. By assumption, the merger persists and faces party L. Notice that the probability that the merged party gets the majority in $t = 2$ is $\Pr\{\xi > \tilde{\xi}_r\} = 1 - F(\tilde{\xi}_r)$ (the same as in the baseline), because the informational cost is only paid by voters in $t = 1$ when the merger is formed. Hence, the expected second period payoff from merging (A-14) is the same as in the baseline model.

In $t = 1$, policy uncertainty introduced by mergers changes how vote shares are computed. Let $v^m_{l,cr,2}$ denote the voter who is indifferent between voting for party L and for a merger among C and R. Formally, $v^m_{l,cr,2}$ solves

$$-\left(v^m_{l,cr,2} - z^m_{cr,2}\right)^2 - \frac{1}{\gamma} + \left(v^m_{l,cr,2} - z_{\ell,2}\right)^2 = 0. \tag{C-5}$$

Solving for the indifferent voter yields:

$$v^m_{l,cr,2} = -\frac{4a^2\left(\gamma\left((\phi+1)^2 - 4z_\ell^2\right) + 4\right) + 4a\gamma(z_\ell - 2)\phi(\phi+1) + \gamma(z_\ell - 2)^2\phi^2}{8a\gamma(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}. \tag{C-6}$$

Using this expression, it is straightforward to compute the vote share of the merged party in $t = 1$:

$$V^m_{cr,1} = \frac{1}{2} + \frac{4a^2\left(\gamma\left((\phi+1)^2 - 4z_\ell^2\right) + 4\right) + 4a\gamma(z_\ell - 2)\phi(\phi+1) + \gamma(z_\ell - 2)^2\phi^2}{16a^2\gamma(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}. \tag{C-7}$$

Differentiating $V^m_{cr,1}$ with respect to γ yields

$$\frac{\partial V^m_{cr,1}}{\partial \gamma} = -\frac{1}{\gamma^2(4az_\ell - 2a(\phi+1) - (z_\ell - 2)\phi)}, \tag{C-8}$$

which is always positive: as γ increases, the uncertainty paid by voter is reduced and the vote share of the merger increases.

Finally, we check if there exists a positive γ such that $V^m_{cr,1} = 1/2$. Solving for γ yields

$$\hat{\gamma} = \frac{16a^2}{4a^2\left(4z_\ell^2 - 1\right) - \phi^2(2a + z_\ell - 2)^2 - 4a\phi(2a + z_\ell - 2)}, \tag{C-9}$$

which is a positive real root. It follows that for $\gamma > \hat{\gamma}$, $V^m_{cr,1} > 1/2$ and the analysis is analogous to the proof of Proposition 4. In particular, let $\Delta_{c,cr} \equiv U^m_{c,cr} - U^{\text{pec}}_{c,cr}$, where

$$U^m_{c,cr} = -(z^m_{cr,1} - z_c)^2 - \sigma^2 - \frac{1}{\gamma} + \delta U_{i,2}(m_{cr}),$$

and

$$U^{\text{pec}}_{c,cr} = -(z^{\text{pec}}_{cr,1} - z_c)^2 - \sigma^2 + \delta U_{i,2}(\neg m).$$

Because uncertainty only affects $\Delta_{c,cr}$ via the term $1/\gamma$, it follows that $\partial(U^m_{c,cr} - U^{\text{pec}}_{c,cr})/\partial\psi$ is always negative, analogously to Equation A-22. Furthermore, for γ big enough, there exists a value of ψ such that $U^m_{c,cr} = U^{\text{pec}}_{c,cr}$, and the result in Proposition 4 continues to hold.

It is left to show that for γ small enough no mergers are sustainable in equilibrium. When $\gamma < \hat{\gamma}$, $V^m_{cr,1} < 1/2$. In this case we have

$$U^m_{c,cr} = -(z_l - z_c)^2 - \sigma^2 - \frac{1}{\gamma} + \delta U_{i,2}(m_{cr}).$$

Note that $U^m_{c,cr} \to -\infty$ as $\gamma \to 0$. This implies that there exists γ' small enough such that $\Delta_{c,cr}(\gamma') = 0$ has no solution. In particular, we have $U^{\text{pec}}_{c,cr}(\gamma') > U^m_{c,cr}(\gamma')$ for all ψ. The analysis for a merger between C and L is analogous therefore omitted. □

Intuitively, Proposition C-1 shows that mergers are only sustainable if they don't introduce excessive uncertainty about where the party platform stands. This can be the case for example when the merged party has a clear statute which is credible given the constituent parties' histories. Low uncertainty can also be a reasonable assumption if constituent parties have been former allies or have had previous experience of governing together. Conversely, Proposition C-1 shows that when voters' uncertainty about the new political party is high, a merger is not a viable alternative to a PEC *even when the electorate is very volatile*.

Appendix D: Pre-Electoral Bargaining

The baseline model assumes that coalition platforms are weighted averages of constituent parties' ideal points, with exogenous weights. This assumption abstracts from a bargaining process over the platform content, which likely takes place among constituent parties before they present their common electoral platform. In what follows, I consider a simple bargaining protocol to show robustness of the main mechanism of the paper.

Recall that ℓ is the closest ideological ally to the centrist party, hence—with exogenous weights—c always prefers ℓ as ally in a one-shot game. The baseline model shows that when the electorate is unpredictable, in equilibrium the centrist party forms a merger as an insurance device against negative shock realizations, and the merger can be formed with the more extreme party r. In principle, if c were allowed to propose a platform to its ally, the reader might expect that the centrist party could choose a sufficiently moderate platform that makes a future r victory unlikely, and such that party ℓ is indifferent between merging with c and running alone.

In fact, I show that this is not the case: in equilibrium, the centrist party sometimes merges with the more extreme r even when coalition platforms are endogenous. In what follows, I assume that constituent parties bargain à la Romer and Rosenthal (1978) and lay out the intuition behind the result. Notice that it is crucial for the result that parties ℓ and r have limited outside options, in the sense that neither can renege in $t = 2$ after a merger has formed in $t = 1$, nor they can form other alliances except with c.

Suppose that ℓ and c merge (form a PEC). The resulting policy platform belongs to the Pareto set of the coalition formed by ℓ and c, i.e.: $z^m_{\ell c} = z^{\text{pec}}_{\ell c} = \beta z_\ell + (1-\beta) z_c$, where the weight $\beta \in [0, 1]$ is *endogenous* and chosen by party c. In particular, c makes a take-or-leave offer to ℓ, which can either accept or reject. Acceptance leads to the formation of a merger. Rejection leads to the following step of the proposal stage, which remains unchanged.

Let us focus on the case where c already has a plurality of votes in the first period. Under this assumption, the formation of mergers in equilibrium is harder to sustain because of c's myopic incentives to run alone, thus it represents a harder test. The following result shows that for high volatility c does merge with the extreme party, and that in equilibrium there are policy concessions ($\beta^* > 0$).

Proposition D-1. *Let $V_{c,1} > \max\{V_{\ell,1}, V_{r,1}\}$, and suppose ℓ is sufficiently moderate. There exists $\hat{\hat{\psi}}$ such that c is indifferent between running alone and merging with ℓ. In equilibrium:*

- *for $\psi < \tilde{\psi}$, a merger between c and r forms, and $\beta^*(\psi, z_\ell) \in (0, 1)$,*
- *for $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, a merger between c and ℓ forms, and $\beta^* = 0$,*
- *for $\psi > \hat{\hat{\psi}}$, parties run alone and $\hat{x}_1 = z_c$.*

Proof. To begin, consider parties' incentives to form alliances in the second period. As in the baseline model, if either ℓ or r have a majority, parties run alone in equilibrium. Suppose instead that ℓ has plurality. In this case, in equilibrium c proposes $\beta^* = 0$ to ℓ, and ℓ accepts. Suppose this is not the case, and that ℓ rejects. Then, c would propose $\beta^* = 0$ to r, and r would accept because the alternative would be z_ℓ implemented by ℓ. Party ℓ is indifferent between $\beta^* = 0$ and the alternative from rejection, i.e., a PEC between c and r with platform z_c. As a tie-breaking rule, I assume that when indifferent parties join the alliance. Finally, because r always accepts a PEC proposal $\beta = 0$, c is indifferent between the two PECs. I assume that when indifferent c proposes a PEC to the closest party ℓ.

It follows that the expected payoff from the second period is (given the uniformity of the shock):

$$U_{i,2}(\neg m_1) = \left[\frac{1}{2} + \frac{\psi}{2}\left(\frac{z_c + z_\ell}{2}\right)\right] u_i(z_\ell) + \frac{\psi}{4}(1 - z_\ell)u(z_c) + \left[\frac{1}{2} - \frac{\psi}{2}\left(\frac{z_c + z_r}{2}\right)\right] u_i(z_r). \quad \text{(D-1)}$$

We can now analyze parties' decision to merge in the first period. From Proposition 2 and 4, we know that there exists a threshold value of volatility ($\tilde{\psi}$) that makes c indifferent between merging with ℓ and r, and that for $\psi < \tilde{\psi}$ ($\psi > \tilde{\psi}$) c prefers to merge with r (ℓ). The difference $U_c^{\text{al}} - U_{c,lc}^m$ simplifies to:

$$\frac{z_\ell\left(\delta\beta\psi - \delta\left(\beta^3 - 1\right)\psi z_\ell^2 - \delta z_\ell\left(\beta^2(\psi + 2) + 2\right) - 4\beta^2 z_\ell\right)}{4}. \quad \text{(D-2)}$$

Differentiating (D-2) with respect to ψ yields

$$\frac{\partial(U_c^{\text{al}} - U_{c,lc}^m)}{\partial\psi} = \frac{\delta z_\ell\left(\beta - \left(\left(\beta^3 - 1\right)z_\ell^2\right) - \beta^2 z_\ell\right)}{4}, \quad \text{(D-3)}$$

which is always positive under the assumptions. Furthermore, there exists a threshold value of volatility ($\hat{\hat{\psi}}$) such that c is indifferent between running alone and merging with ℓ, where:

$$\hat{\hat{\psi}} = -\frac{2z_\ell\left(2\beta^2 + \delta\left(\beta^2 - 1\right)\right)}{\delta\left(\left(\beta^3 - 1\right)z_\ell^2 + \beta^2 z_\ell - \beta\right)}. \tag{D-4}$$

I consider the same equilibrium of the baseline model as the candidate equilibrium: c proposes merger to r (ℓ) for $\psi < \tilde{\psi}$, a merger to ℓ for $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, and parties run alone for $\psi > \hat{\hat{\psi}}$.[23]

(i) Suppose $\tilde{\psi} < \psi < \hat{\hat{\psi}}$ (such that c prefers a merger with ℓ). Can c offer $\beta = 0$ (i.e., its preferred policy)? Party ℓ's incentive compatibility condition for accepting a merger (when c has plurality) is:

$$\frac{1}{4}z_\ell\left(\delta\left(\left(-\beta^3 + 2\beta^2 + 1\right)\psi z_\ell^2 - \left((\psi+2)\beta^2 - 4\beta + 2\right)z_\ell + (\beta-2)\psi + 4\right) - 4z_\ell(\beta-2)\beta\right) > 0. \tag{D-5}$$

Substituting $\beta = 0$, The IC condition above simplifies to

$$\frac{1}{4}\delta z_\ell\left(-2\psi + \psi z_\ell^2 - 2z_\ell + 4\right) > 0, \tag{D-6}$$

where the sign of the LHS depends on the value of z_ℓ and ψ. In particular, offering $\beta = 0$ is only incentive compatible for ℓ for high values of ψ, so that the LHS is positive. Are other values of β sustainable? As β increases, the merger platform shifts to the left, thus losing

[23] Notice that the value of $\tilde{\psi}$ is equal to the baseline model, whereas the value $\hat{\hat{\psi}}$ differs because of the different continuation value of not merging in $t = 1$.

moderate votes to r. Furthermore, as the platform weight of ℓ increases, all else equal c prefers to run alone, thus c's incentive compatibility condition becomes more binding.

ii) Suppose $\psi < \tilde{\psi}$ (such that c prefers a merger with r). Can c offer $\beta = 0$ to r? Party r's incentive compatibility condition for accepting a merger is:

$$\frac{1}{4}\delta\left(\beta^3\psi - 2\beta^2(\psi+1) + 4\beta - \psi - (\beta-2)\psi z_\ell^2 + z_\ell\left(\beta^2\psi+4\right) - 2\right) - (\beta-2)\beta > 0, \tag{D-7}$$

which is not satisfied when $\beta = 0$ for high electoral volatility ($\psi < \tilde{\psi}$), that is precisely when c would like to merge with r (rather than ℓ). Thus, for this range of volatility c needs to offer r a platform β^* that makes r indifferent between merging and running alone. It is possible to show that such β^* exists (the expression is lengthy and does not provide further intuition, and therefore I omitted it from the proof), and is such that $\beta^* \in (0,1)$ for $\psi < \tilde{\psi}$. Finally, notice that:

$$\frac{\partial(U^m_{r,cr} - U^{\text{al}}_r)}{\partial\beta} = -\frac{1}{4}(z_\ell - \beta)(\delta(3\beta\psi + \psi z_\ell - 4) - 8) < 0. \tag{D-8}$$

This implies that any $\beta > \beta^*$ is accepted by r. For any such β, an equilibrium where the centrist party merges with the extreme party r can be sustained. □

As in the baseline model, party c prefers to merge rather than running alone for high values of volatility, because mergers represent an insurance against negative shock realizations. Suppose that, when $\tilde{\psi} < \psi < \hat{\hat{\psi}}$, c proposes a merger to ℓ with platform $\beta z_\ell = 0$. Figure 4 illustrates the values of ψ, β such that c and ℓ prefer to merge than running alone. As the left panel of Figure 4 illustrates, c (ℓ) prefers a merger to running alone for high (low) volatility, and there exists a parameter configuration such that a merger with platform equal to z_c forms.

Are other values of β sustainable? As β increases, the merger platform shifts to the left, thus losing moderate votes to r. Furthermore, as the platform weight of ℓ increases, all else equal c prefers to run alone, thus c's incentive compatibility condition becomes more binding, as shown in the right panel of Figure 4.

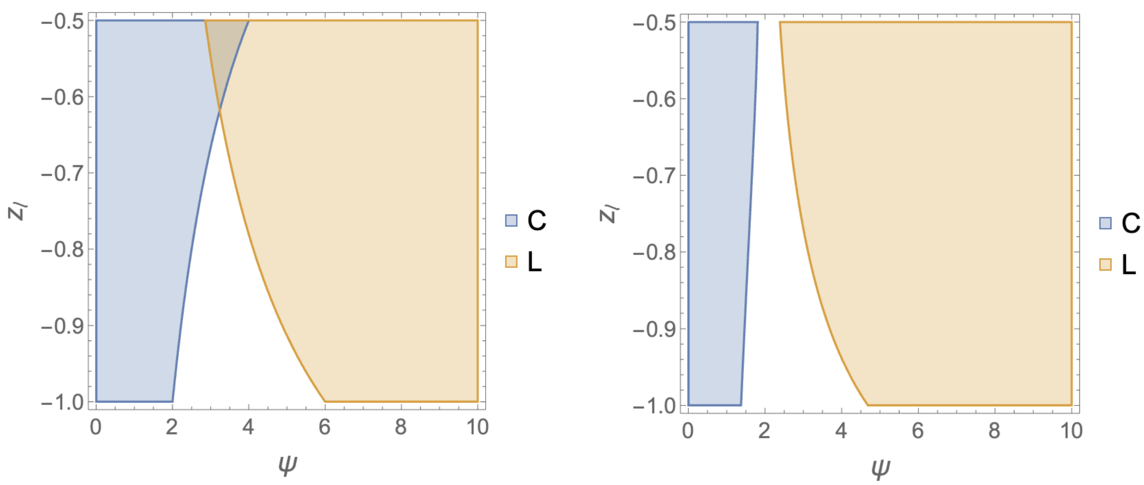

Figure 4 – Values of ψ (x axis) and z_ℓ (y axis) sustaining a merger between ℓ and c in equilibrium for $\delta = 0.7$. In the blue region, a merger is incentive compatible for c. In the orange region, a merger is incentive compatible for ℓ. In the left panel, $\beta = 0$. In the right panel, $\beta = 0.2$.

Suppose $\psi < \tilde{\psi}$ (such that, from Proposition 2, c prefers a merger with r). For this range of volatility c needs to offer a platform β^* that makes r indifferent between merging and running alone. When β is low, a merger is not incentive compatible for r, which all else equal prefers a higher weight on its preferred policy in the merger platform. As β increases, r's incentive compatibility constraint relaxes and r is willing to merge for more values of z_ℓ and ψ. Intuitively, a higher weight in the merged platform trumps the advantages of running alone. As β increases, however, party c is worse off and less likely to merge with r. Figure 5 shows that there exist parameter values such that an equilibrium where the centrist party merges with the extreme party r can be sustained. In the left panel, $\beta^* = 0.25$ and a merger is not incentive compatible for r. In the right panel, where $\beta^* = 0.45$, r's incentive compatibility constraint relaxes and r is willing to merge for more values of z_ℓ. Otherwise, r is better off running alone.

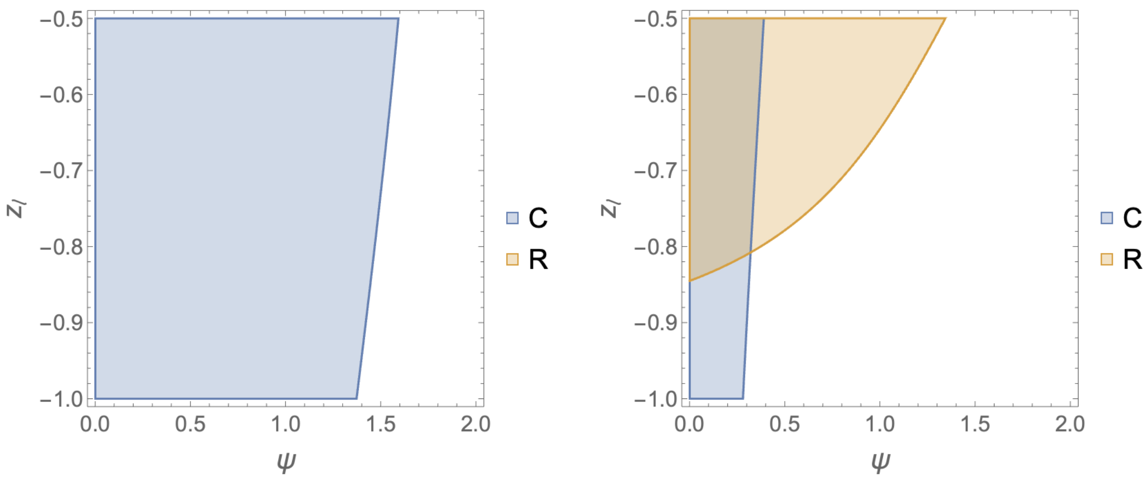

Figure 5 – Values of ψ (x axis) and z_ℓ (y axis) sustaining a merger between c and r in equilibrium for $\delta = 0.7$. In the blue region, a merger is incentive compatible for c. In the orange region, a merger is incentive compatible for r. In the left panel, $\beta = 0.25$. In the right panel, $\beta = 0.45$.