Evolving Parties

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Abstract

This paper explores the reasons why factions within political parties might choose to split and when instead party unity is expected. We develop a theory based on the premise that political factions aim to preserve and cultivate their individual brands. In the model, two factions can belong to the same party or not. When together, factions can decide to split. A split sets in motion the evolution of factions' individual brands, which can be positive or negative. When apart, factions can decide to merge. By merging, factions reap a benefit from being together, but need to divide party resources according to their relative strength. We characterize when splits and mergers are stable—reflecting fragmented and non-fragmented party systems respectively—and when instead cycles emerge in equilibrium. Factions may want to split even if by doing so they hurt their brand. These damaging splits, we show, only happen when factions re-merge in the future: by merging, the splinter faction might gain either by becoming the bigger fish in a smaller pond or the smaller fish in a bigger pond.

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1. Introduction

As the literature increasingly acknowledges, political parties are not homogeneous entities and often consist of different factions, which hold varying beliefs and priorities. These factions can be based on ideology, geography, ethnicity, religion, or other factors that distinguish them from one another. In turn, these divisions can lead to conflict and competition within the party, resulting in intense debates and negotiations, as well as a struggle for power and influence. Sometimes, these disagreements between factions within a party can even lead to a split.

In some cases, factional splits are easy to understand as the newly formed splinter party goes on to be very successful, attract supporters from the original party and potentially cause a shift in the balance of power. Consider for example the history of the UK Labour party. In the 1980s, a group of centrist Labour Party MPs broke away and formed the Social Democratic Party, citing concerns over the party's leftward shift. The split ultimately led to the formation of the Liberal Democrats, which grew to be an important political force in the UK.

Other cases are more puzzling, as the splinter party fails to gain significant support or influence. For example, in 2017 Pierluigi Bersani, former leader of the Italian *Partito Democratico* (PD), resigned as party leader and, along with a group of like-minded politicians, founded a new political party, *Articolo 1*. The new party positioned itself as a left-wing alternative to the more centrist PD. The split had significant repercussions for both the PD and the new *Articolo 1*. While the PD lost a number of its more left-leaning members to the new party, weakening its position in Italian politics, the newly founded *Articolo 1* struggled to gain traction and failed to make a significant impact in subsequent elections.

This paper presents a theory of party evolution that focuses on factions' incentives to stay together or apart. Our theory can account for both intuitive cases of party splits where the splinter party is successful and for puzzling cases whereby the new party fails to gain electorally. Additionally, we examine the circumstances under which parties may opt to remain intact rather than breaking apart, or when different parties may merge to form a new political entity. By analyzing the endogenous evolution of parties, we provide a new perspective on party system change that emphasizes its supply-side.

Our theory is predicated on the assumption that each faction within a political party is characterized by a specific brand, reflecting its ideology, resources, and electoral support. For example, in the United Kingdom, the Labour Party is a center-left political party that includes various factions with different brands, such as the Blairite faction associated with former Prime Minister Tony Blair, the Corbynite faction associated with former leader Jeremy Corbyn, and the more centrist faction associated with current leader Keir Starmer. These factions are not just defined by their ideology or policy positions, but also by their resources and organizational capacity. In many cases, they have their own networks of donors, activists, and media outlets, which can help them to gain more influence within the party.

In this perspective, what we refer to as a faction brand determines its expected success if the faction decides to split and run alone, as well as its relative power within the party. As such, a primary consideration for a faction is the need to preserve and cultivate its own brand. Our model allows us to analyze how such strategic considerations influence the stability or instability of political parties.

Formally, we study the repeated interaction between two factions that belong to the same ideological camp. In each period in which factions are together, they can unilaterally decide to split and form a separate party. In each period in which factions are split, they can merge again if they both agree to do so. Each faction is characterized by a brand, reflecting its resources and electoral support. Thus, a faction's brand evolves depending on its decision to split or merge in each period. In particular, as highlighted by the examples mentioned above, a split may damage a faction's brand or may boost it.

Whether they split or stay together, the factions' brands determine their payoff. When factions split, their brand determines their electoral strength. When they are together, their brand contributes to the strength of the party *and* determines the internal division of the spoils. Further, when factions are in the same party they enjoy an efficiency gain, i.e., the party's strength is more than the sum of the individual factions'. For instance, we could think of this gain as a consequence of institutional factors such as the electoral system's disproportionality, which incentivizes factions to stay together.

Our analysis uncovers a rich set of equilibria. Intuitively, if splits are too damaging to factional brands (perhaps because voters punish a divided camp), in equilibrium we have a stable party with no splits. In contrast, if a faction benefits a lot from a split (perhaps due to visibility gains or because its ideology is more appealing to voters), then we will have a split at the beginning of the game, with this fragmented system stable over time.

Suppose instead that the impact of a split on the factions' brands is not too detrimental or too beneficial. Here, we find that the equilibrium must exhibit cycles. The two factions (which belong to the same party at the beginning of the game) split today only to re-merge tomorrow.

Interestingly, these splits and re-mergers do occur in the real world. Years after the damaging split described above, the Italian party *Articolo 1* merged back together with the PD on June 10th 2023, joining the party revitalized by the leadership of its new secretary Elly Schlein. The Norwegian Liberal People's Party was a social liberal political party established by a split in the Liberal Party over the issue of Norway's accession to the European Economic Community in 1972. In 1988, the party officially merged back together with the Liberal Party. In the Netherlands, the Catholic National Party was founded in 1948 by Charles Welter, then minister belonging to the Catholic People Party, as a protest against Indonesian independence. The party eventually re-merged with the Catholic People Party in 1955. In the United States, the Progressive Party was formed in 1912 after a split from the Republican Party. The Progressive Party was led by former President Theodore Roosevelt, who had been dissatisfied with the conservative policies of his successor, William Howard Taft. However, after the Progressive Party lost the 1912 presidential election, many of its members rejoined the Republican Party.

In line with these examples, we find that in our model a cycling equilibrium can emerge even (yet not only) if the splinter faction's brand is *damaged* by the split in the first period. To see why, suppose that splitting damages both factions' brands. Then, when the factions reunite in the second period, the party is weaker than if it had remained unified. However, one faction may still choose to split if it believes that doing so will cause more harm to its opponent. This is because, by re-merging after splitting, the splinter faction expects to obtain a larger portion of a smaller pie. Therefore, the splinter faction is prepared to incur a cost in the present to improve its relative position within the party, even if it harms the party as a whole.

One might think that the trade-off of splitting versus staying together in the first period applies only to the relatively stronger faction (whose brand is evolving more), and that the weaker faction always prefers to remain in the party having nothing to gain from a split. Perhaps surprisingly, we show that splits could also be initiated by the weaker faction, as part of an equilibrium cycle. This happens when the cycle statically harms the splinter faction, but benefits the other one and the entire party. Here, the splinter faction is willing to split immediately so that the party can become stronger in the future. This is done in anticipation of reuniting in the future to enjoy the gains, which will be a smaller portion of a larger pie.

The existence of a cycle equilibrium sustained by the first, 'bigger fish in smaller pond' dynamic generates interesting comparative static results. We show that, under some conditions, increasing the efficiency premium the factions gain when they stay together (e.g., increasing the disproportionality of the electoral system) *encourages* a split. If a faction can initiate a split today expecting to re-merge tomorrow from a stronger bargaining position within the party, increasing the value of the pie will only strengthen its incentives to split. Following a similar logic, making the party organization less egalitarian (or increasing ideological divisions in the party) will only encourage a split initiated by the faction expecting to become a bigger fish in a smaller pond. This happens because the cycle is sustained by the splinter's incentive to become stronger, and a less egalitarian internal organization sharpens this incentive.

In concluding this section, let us emphasize that we are not implying that the evolution of political parties is solely based on individual factions' incentives to cultivate their brand. Indeed, existing research has emphasized the role of institutional factors such as electoral systems (Strom, Budge and Laver, 1994; Kaminski, 2001; Golder, 2006*a*,*b*; Clark and Golder, 2006; Blais and Indridason, 2007), and voter preferences (Rokkan and Lipset, 1967; Dalton and Flanagan, 2017; Pedersen, 1979; Shamir, 1984; Taagepera and Grofman, 2003; Kuenzi and Lambright, 2001; Coppedge, 1998; Birch, 2003) as drivers of party system dynamics. Thus, our contribution is to offer a new perspective that complements existing research on institutional and voter-driven factors, and focuses on the internal dynamics of political parties. By highlighting the importance of factional brand cultivation, we therefore offer new lens through which understand the evolution of political parties.

2. Literature Review

Our theory is based on the premise that parties are not monolithic entities, but are internally divided into competing factions. The formal literature has increasingly acknowledged the importance of factions to understand political parties' nomination processes (Caillaud and Tirole, 2002; Crutzen, Castanheira and Sahuguet, 2010; Hirano, Snyder Jr and Ting, 2009), intra-party power sharing (Invernizzi, 2022; Invernizzi and Prato, 2024), and intra-party competition, both over resources (Persico, Pueblita and Silverman, 2011) and ideology (Izzo, 2023). We share with this literature the focus on within-party actors, political factions. We show how considering factional incentives to develop their brand leads to unexpected predictions on party evolution.

The literature of American and comparative politics has put forward a few alternative hypotheses for why parties emerge and change. One approach focuses on the voter demand side as the key explanation for party emergence. According to the primordialist account (Rokkan and Lipset, 1967), parties originate as a consequence of social cleavages in societies. The more numerous the cleavages, the higher the number of parties, and new parties emerge as a consequence of new cleavages. An opposite "top-down" approach is the one taken by Downs (1957) and subsequently revisited by Aldrich (1995), according to whom parties are set in motion by career concerned politicians who need an institutional machinery to support them in elections and once in office.

Our paper provides a rationale for political parties that focuses on the cultivation of their brand. As such, it is primarily related to the formal literature that focuses on top-down, elite-based explanations for party formation. In particular, Snyder and Ting (2002) study how endogenous platforms, or brands, allow candidates to signal their preferences to voters. Levy (2004) analyses party formation in the presence of a multidimensional policy space, where policy-motivated politicians can form coalitions (parties) to credibly commit to a broader set of policies (the Pareto set of the coalition). While these works analyze policy outcomes resulting from parties, we study when parties strategically decide to form (and dissolve), in expectation of changes in their brand — which captures their underlying electoral strength.

By analyzing the endogenous evolution of parties, this paper provides a novel theoretical perspective on party system change. Most of the empirical literature on party system evolution focuses on voters as the main driver of stabilization, both in Western countries (Dalton and Flanagan, 2017; Pedersen, 1979; Shamir, 1984; Taagepera and Grofman, 2003) and in new democracies such as Africa (Kuenzi and Lambright, 2001), Latin America (Coppedge, 1998) and Eastern Europe (Birch, 2003). In these accounts parties are primarily by-products of pre-existing societal cleavages, and instability in party system results from instability in such cleavages or voter preferences (Tavits, 2008). In contrast, our model highlights that strategic elites might also affect the stability of the party system (Cox, 1997; Kitschelt et al., 1999; Tavits, 2008; Invernizzi, 2023), and that this could happen in unexpected ways. Here, the crucial contribution is that factions' dynamic considerations may give rise to party system instability (through cycles) when static consideration of voter preferences would induce stability.

Methodologically, this paper develops a dynamic theory of party evolution. Related models of party system change have typically focused on *party entry* as a determinant of change. For instance, Buisseret and Van Weelden (2020) study how an outsider candidate decides to enter the electoral contest (either via primaries or via a third-party), while Kselman, Powell and Tucker (2016) focus on party entry in Proportional Representation systems. In contrast, we focus on the determinants of party system change that arise from the incentives of existing parties to change their organization by splitting and merging.

3. A model of Party Evolution

We study the interaction between two factions, i and j, belonging to the same (left-wing) ideological camp. The game has two periods, $t = 1, 2$. At the beginning of the game, the two factions are together in the same party, so the party and camp coincide. In each period in which factions are together, they can unilaterally decide to split and form a separate party. In each period in which factions are split, they can merge again if they both agree to do so. For clarity of exposition, we begin by assuming that only one of the factions (say faction i) can initiate a split. Below, we analyze an extension where both factions are allowed to split.

Factions' Brands. In each period, each faction is characterized by a brand, b_t^i and b_t^j, reflecting the faction's ideology, resources and electoral support. The evolution of the factions' brands depends on the decision to split or merge in a given period. We begin by assuming that factions have the same initial 'stock' of brand: $b_0^i = b_0^j = b_0 > 0$. If factions remain merged in the same party, then their first-period brands are fixed at b_0. Instead, after a split the brands of the two factions begin evolving independently. Suppose there is a split in period 1, following faction i's unilateral decision to exit the party. Then, the factions' brands in period 1 are

$$b_1^i = b_0\ k^i \quad \text{and} \quad b_1^j = b_0\ k^j, \tag{1}$$

where $k^i > 0$ and $k^j > 0$ determine the per-period shift in each faction's brand.[1] If $k^i > 1$, then a split helps faction i to build its own brand. For example, splitting might help a faction's members getting more exposure, and a split helps directing more resources into a faction's brand. Alternatively, the splinter faction may be able to establish its new ideological identity as one that is more appealing to voters. If instead $k^i < 1$, then a split damages the faction's individual brand. This might reflect voters' perception of the splinter party as too extreme, the resource loss from leaving the party, or even voters reacting negatively to the split itself.

If factions remain split for both periods, then the brands continue evolving independently. The brands in period 2 are then

$$b_2^i = b_0\ (k^i)^2 \quad \text{and} \quad b_2^j = b_0\ (k^j)^2. \tag{2}$$

If factions decide to re-merge in period 2, then their second-period brand is instead

$$b_2^i = b_1^i \quad \text{and} \quad b_2^j = b_1^j, \tag{3}$$

that is, the faction's brand ceases to evolve in the period the merger is formed. Thus, a merger 'freezes' the two factions' relative brands. Similarly, if factions never split then we have

$$b_2^i = b_1^i = b_0 \quad \text{and} \quad b_2^j = b_1^j = b_0. \tag{4}$$

[1] Of course, it could be that shifts in factions' brands differ across periods. While we model these shifts as time-independent, our results would be qualitatively unchanged if we allowed for the ks to evolve over time.

Payoffs. A faction's brand influences its payoff not only when it decides to run independently but also when it operates within the larger party structure. Formally, let u_t^i be faction i's payoff in period t. Then

$$u_t^i = \begin{cases} \left(b_t^i + b_t^j + a(r_t)\right) \left[\frac{1}{2} + \phi(b_t^i - b_t^j)\right] & \text{if in the same party in } t \\ \\ b_t^i + \eta r^t & \text{if running alone in } t \end{cases} \tag{5}$$

The utility u_t^j for faction j is defined symmetrically.

When factions remain within the same political party, their payoff is commonly linked to the overall strength of the party $\left(b_t^i + b_t^j + a(r_t)\right)$. However, the distribution of rewards among factions is influenced by the relative strength of their respective brands.

The strength of the party can be understood as a function of the strength of the brand of the two factions, represented by b_t^i and b_t^j respectively, as well as an exogenous component that captures the overall strength of the ideological camp in a given time period, denoted by $a(r_t)$. In the baseline model, we will set $a(r_t) = \alpha r^t$. It is important to note that both α and r are positive, with r reflecting the shifting preferences of the electorate, which can either support or oppose the ideological camp. When r is greater than 1, it means that the camp is gaining support over time, whereas when r is less than 1, it implies that the camp is losing support.

The internal allocation of resources among factions is determined by their relative brand strengths. In our model, the parameter $\phi \geq 0$ represents the elasticity of payoff in relation to the relative strengths of the factions when they are united within the same political party. When $\phi = 0$ the factions always share the pie equally, regardless of their relative brands. When $\phi > 0$, instead, the faction with the strongest brand gets a larger share of the pie. This parameter thus captures the internal organization of the party and highlights the costs faced by the weaker faction and the advantages gained by the stronger one. Thus, higher ϕ captures a less egalitarian party organization, where the distribution of rents is highly dependent on the factions' relative strength. Alternatively, higher ϕ could represent stronger ideological divisions within the party, that make it even more costly for the weaker faction to maintain its position within the party.

Finally, ϕ can capture the intensity of the factions' rent-seeking motivations that push them to strengthen their own bargaining position within the party, even absent ideological divisions.

Note that the parameter ϕ is exogenous in our model. However, it could potentially be microfounded as the outcome of an internal power struggle between the factions. Previous studies have explored this possibility and have shown that internal contests for power can significantly impact the distribution of resources within a political party (Invernizzi, 2022; Invernizzi and Prato, 2024). Additionally, note that ϕ is appropriately bounded to ensure that a faction's share of rents is between 0 and 1. In particular, we assume that $k^i, k^j \in [0, \bar{k}]$, and $\phi < \frac{1}{2\bar{k}^2}$.

Suppose instead that the factions split in period t, so that i's payoff is $u_t^i = b_t^i + \eta r^t$. When a faction runs alone, its strength is determined by its own brand and the strength of the camp as a whole. Obviously, when running alone each faction gets to keep the entirety of the pie it gains. We will make the assumption that $\alpha > 2\eta$ in this context, to represent the efficiency premium that results from the two factions joining forces. In other words, the overall strength of the party is more than the sum of the individual components. To reduce the number of parameters to consider, we will set η to zero.[2] In this setting, the parameter $\alpha > 0$ could represent institutional factors, such as the degree of disproportionality in the electoral system, creating economies of scale that incentivize factions to stay together. In a highly disproportional electoral system, for example, the advantage of belonging to a larger party is significant, as the party's combined vote share translates into an even larger number of seats. This is then captured by a larger value of α.

The history of the game at the beginning of period t (h_t) is the list of split/merge decisions up to period t. A strategy for faction i determines i's action after every possible history. Formally, given a history h_t, faction i decides in period t whether to split or merge. We focus on subgame-perfect Nash equilibria of the game.

Discussion of the Assumptions

In our model, k^i and k^j represent, in reduced form, how voters' react to factional splits. Notice that we allow, in principle, both factions to be damaged by a split, or both to profit from it.

[2] Even if we set η to zero, it is useful to introduce this parameter in the utility function to show the different components of the overall strength of the ideological camp in (5) to show the effect of splitting on the multiplier coefficient.

The underlying assumption is that the electoral performance of the two factions is not the result of a zero-sum game: each faction may mobilize different groups of voters, or lose their support. Rather than assuming that voters are motivated solely by ideological proximity, the advantage of our modeling assumption is that it allows to capture in a parsimonious way the richness of voters' attitudes and behavior. This approach then allows us to have a setup that captures several cases of interest.

As a potential microfoundation of our assumptions, imagine that at the onset of the game each faction's base of potential supporters is composed of three groups. First, a solid core, that mobilizes whether factions are together or apart. Second, a group of potential abstainers, that may fail to mobilize if dissatisfied with the faction's image or actions after a split. For example, the faction's leadership may undergo more intense scrutiny after a split, and may fail to persuasively mobilize these voters. Alternatively, the faction may appear as too extreme or incapable after a split. Importantly, it needs not be the case that all the voters in this group would support the opposing faction if they fail to be mobilized by the splinter. Finally, and symmetrically to the previous group, the third group contains voters that may be mobilized when a faction splits from the party. For example, the splinter faction might focus on supporting an issue relevant to a particular constituency that would not vote for the old party otherwise. Alternatively, the faction's leadership may benefit from the increased media attention following a split, and use it to attract support from this group. As above, it needs not be the case that all the voters in this group are 'stolen' from the opposing faction's potential supporters.

Thus, we may have a situation where both factions are damaged by a split, $k_i < 1$ and $k_j < 1$. Both may benefit, $k_i > 1$ and $k_j > 1$, or the effect may be different for the two factions, e.g., $k_i > 1$ but $k_j < 1$. This is in addition to the efficiency premium, which captures the observation that when running together the factions benefit from voter coordination or the disproportionality of electoral rules.

Finally, let us highlight that in our setup factions face no uncertainty over the consequences of a split for their relative brands (i.e., k^i and k^j are known). We impose this assumption in order to more clearly illustrate the mechanism behind the results, and show that dynamic incentives may generate splits in equilibrium even if factions can perfectly anticipate that this will be costly in the short run (i.e., the split is statically damaging). It is intuitive that introducing a small amount of uncertainty (e.g., assuming the realized k^i and k^j are uncertain, but the players' priors are sufficiently precise) would not alter our qualitative conclusions. In concluding the paper, we then briefly discuss how a large amount of uncertainty (e.g., considering priors with a high variance) may enrich our dynamics.

4. Analysis

We assume that both factions are already merged into a single party at the outset of the game. The aim of the analysis is then to examine how this party evolves over time. Specifically, we seek to identify the conditions under which the following scenarios occur:

1. A stable, non-fragmented party, where the factions remain merged in both periods;
2. An unstable party, where the factions remain merged in the first period but split in the second;
3. A stable fragmentation, where the factions split in the first period and do not re-merge;
4. Cycles of fragmentation, where the factions split in the first period and subsequently re-merge in the second.

Before fully characterizing the equilibrium, let's examine the strategic incentives that factions encounter in this setting. To do so, it is useful to start from a *static* benchmark. Suppose that factions only consider their current period payoff. Assuming for simplicity that faction j would agree to a merger (or that factions are already merged at the beginning of the period), i would choose to split in period t if and only if

$$b^i_{t-1}\, k^i > \left(b^i_{t-1} + b^j_{t-1} + \alpha r^t\right)\left[\frac{1}{2} + \phi(b^i_{t-1} - b^j_{t-1})\right], \tag{6}$$

where b^i_{t-1} is either b_0 or $b_0 k^i$ (and analogously for b^j_{t-1}) depending on the t and the history leading up to it.

The faction's incentives in this static benchmark are quite straightforward. If k^i is large enough, the faction will gain a lot from running alone and will therefore choose to split from the party. On the other hand, if k^i is small, the faction will choose to remain within the party. It is important to note that when $b_t^i = b_t^j$ (as is the case in our setting in period 1), a faction would never choose to split when $k^i < 1$. This is because damaging splits hurt the faction in the period of the split, hence would not emerge in equilibrium with myopic factions.

The comparative statics in this benchmark case are also intuitive. A larger α signifies a higher efficiency premium from the factions staying together, making it easier to maintain the equilibrium where the party stays united. Conversely, intensifying internal divisions or making the party organization less egalitarian (i.e., increasing ϕ) always incentivizes the weaker faction to split.

In our analysis below, we will see that while some of these intuitive results hold for forward-looking factions, others need to be qualified. In particular, we will show that damaging splits can emerge in equilibrium: dynamic incentives may push a faction to split even when this behavior would never be statically optimal (i.e., when $k^i < 1$). Furthermore, the comparative statics results are richer, and under some conditions go in the opposite direction of what described above.

For clarity of exposition and to reduce the number of cases under consideration, but without much loss of generalityfor our qualitative results, we will assume that $r \geq 1$, so that the ideological camp is (weakly) gaining support over time. It is easy to prove that this implies that we can never have an equilibrium where the factions stay merged in the first period but split in the second, since the efficiency gain from being in the same party increases over time.

5. Equilibria

We now go back to the assumption that factions are forward-looking. Under $r \geq 1$, we have:

Proposition 1. *There exist unique $\underline{k}^i \leq \overline{k}^i$ such that, in equilibrium:*

- *The factions remain merged in both periods if $k^i < \underline{k}^i$;*
- *The factions split in the first period and remain split if $k^i > \overline{k}^i$;*
- *Finally, if $k^i \in [\underline{k}^i, \overline{k}^i]$, the factions split in the first period and re-merge in the second.*

The first and second bullet-points are intuitive, and mirror the static benchmark. When k^i is sufficiently large, i can run alone and avoid the need to share the pie while also cultivating its brand. Thus, the party splits in the first period and remains split. In contrast, when k^i is too low a split is too damaging (or not sufficiently beneficial to compensate for the loss of the efficiency premium αr^t). Factions prefer to stay in the same party enjoying the efficiency gains from being together, and the party remains united for both periods.

More interesting, we see that for intermediate values of k^i, a cyclic equilibrium can emerge, in which the factions begin the game united, then split in the first period, only to re-merge in the second. Importantly, this cycle does not emerge because factions face uncertainty over the consequences of different strategies, nor because the *statically* optimal strategy changes over time.

This is evident from the fact that, as depicted in Figure 1 below, for some values of r this cycle equilibrium is sustainable when $k^i < 1$, i.e., when the split always hurts the faction's static payoff. Put differently, the emergence of the cycle equilibrium is not driven by the faction's short-term gains or static considerations. Rather, it reflects a dynamic strategic behavior where faction i expects that splitting in the first period will pave the way for a future, more advantageous re-merger in the second period. As a result, faction i may find it beneficial to split, even if this entails a static loss in the short run. Of course, for the second-period merger to be profitable for i in the second period k^i cannot be too large. At the same time, k^i cannot be too small, to ensure that the static cost of the split in the first period is not too high.

Further, notice that for the cycle to be sustained, j must be able to credibly commit to merging in the second period. This Corollary follows straightforwardly:

Corollary 1. *There exists a unique* $\widetilde{k}^j$ *s.t.* $\underline{k}^i < \overline{k}^i$ *only if* $k^j < \widetilde{k}^j$.

5.1. The Logic of Cycles and Damaging Splits

Having establish our equilibrium characterization, we now delve deeper into the logic underlying the emergence of cycles of mergers and splits. To provide a clearer understanding of the dynamics that give rise to this cycle equilibrium, we will specifically examine the scenario where $k^i < 1$, which results in a statically damaging split in the first period. As we will see, there are two

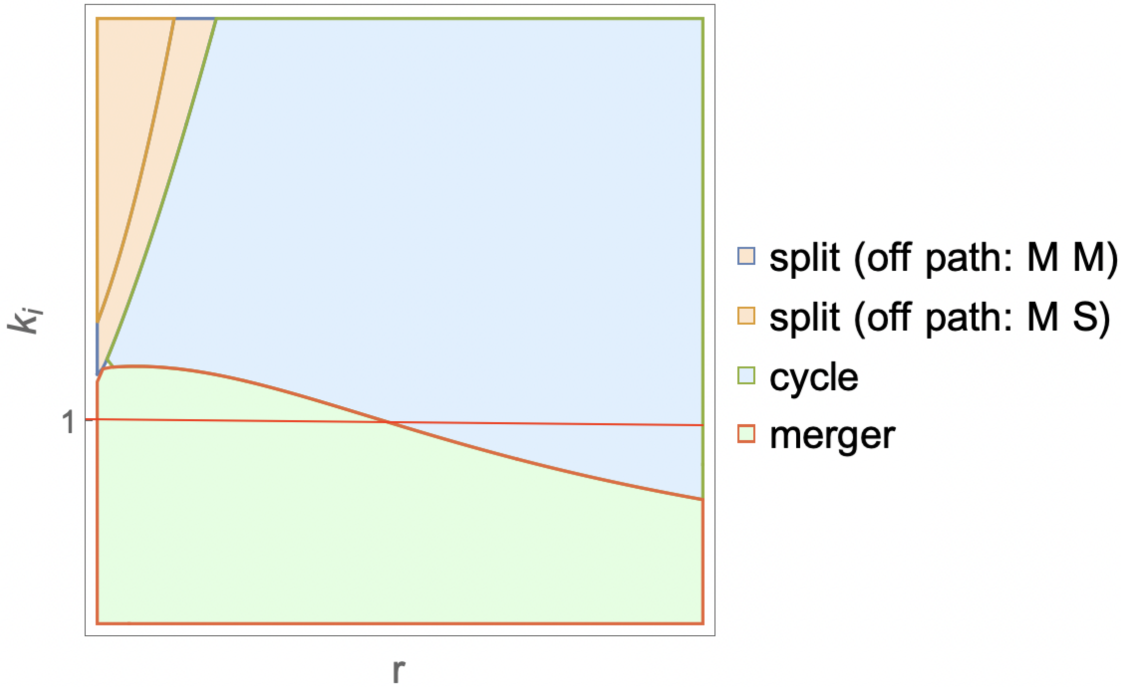

Figure 1 – Equilibria illustration for $k^i \in [0.5, 2]$, $r \in [2, 18]$. The orange region correspond to the stable split equilibrium, the blue region to the cycling equilibrium, and the green region to the stable merger equilibrium. The other parameters are set to $\alpha = 1$, $k^j = 0.5$, $b = 1$, $k^j = 0.5$.

possible types of dynamics that may drive this cycle, depending on whether $k^i > k^j$ or $k^i < k^j$.

1. $k^i > k^j$**: bigger fish in smaller pond.** First, suppose that a split damages *both* the splinter faction and the party as a whole: i.e., it depletes both factions' brands ($k^i, k^j < 1$). Here, when the factions reunite in the second period, the party will be weaker than it would have been without the split, and faction i's own brand will be less valuable. Nonetheless, it can still be advantageous for faction i to instigate the split if it inflicts even greater damage on the opposing faction, j (i.e., $k^j < k^i$). In this case, even if i's *absolute* brand will have weakened, its position *relative* to the opposing faction j will have strengthened.

In other words, even if the party becomes weaker as a result of the cycle, faction i chooses to split today to enhance its standing in the party in the future. This decision is made precisely because faction i expects to merge back and reap the benefits of a weakened opposing faction:

grabbing a bigger share of a smaller pie.

2. $k^j > k^i$: smaller fish in bigger pond. Next, suppose $k^i < 1 < k^j$, meaning that the split is damaging to the splinter faction's brand but improves the opposing faction's. In this case, the cost of the split is very high for faction i. Not only it imposes an immediate cost in the first period, but it also puts the faction in a weaker bargaining position within the party following the re-merge in the second period.

Although the cycling equilibrium may seem counterproductive, it can still be sustained under certain circumstances. Specifically, if $k^i < 1 < k^j$ and $k^i + k^j > 2$ the cycle damages the splinter faction but ultimately helps the camp overall. Thus, when the party re-merges in the second period it is stronger than it would have been absent the split. In this scenario, faction j benefits from the cycling equilibrium since it strengthens both the camp and its relative power. Meanwhile, the splinter faction i is willing to incur the cost of splitting today because it expects that doing so will make the party stronger tomorrow. Again, i chooses to split today precisely because it anticipates re-merging tomorrow, and thus ultimately benefits from grabbing a smaller share of a bigger pie.

Finally, note that cycles can never emerge when $k^i < k^j < 1$: in this case a split damages the camp as a whole, hurts factions in the short run *and* damages the splinter faction's standing within the party. It is then straightforward that faction i never finds it profitable to split and re-merge in this case.[3]

6. Comparative Statics

The different logic sustaining the cycle under $k^i > k^j$ or $k^i < k^j$ also emerges in our comparative statics results:

Proposition 2. *Suppose that* $\underline{k}^i < \overline{k}^i$. *Then,*

(i) For $k^i > k_j$, $\overline{k}^i$ *is increasing and* $\underline{k}^i$ *is decreasing in* ϕ*;*

(ii) For $k^i < k_j$, $\overline{k}^i$ *is decreasing and* $\underline{k}^i$ *is increasing in* ϕ*.*

[3] However cycles could still happen under $k^i < k^j < 1$ if both factions could initiate cycles: we discuss this scenario in a separate section below.

A larger ϕ captures a less egalitarian internal organization, with more resources going to the strongest faction, and/or a party platform that reflects less the ideological preferences of smaller factions. As such, naive intuition would suggest that increasing ϕ should always make a stable merger equilibrium harder to sustain, because weaker factions' participation constraints are harder to satisfy under a less egalitarian organization, hence factions might split. This intuition is validated in our model in the case of $k^i > k^j$, but not when $k^i < k^j$.

Recall that under $k^i < k^j$ if a cycle emerges it is sustained by a "smaller fish in bigger pond" dynamic. The splinter faction is willing to pay an immediate cost and damage its standing within the party, in order to strengthen the party's position. This dynamic is more profitable for the splinter faction as ϕ decreases: when the party is more cohesive or has a more egalitarian structure, i's cost of damaging its relative bargaining power (i.e., reducing $b^i_t - b^j_t$) is reduced. Thus, as ϕ increases the cycle equilibrium is harder to sustain, while the stable merger is easier to maintain.

In contrast, when $k^i > k^j$ the incentives underlying the cycle is for i to strengthen its relative standing within the party. As ϕ increases these incentives become stronger. The cycling equilibrium region expands, eroding both the stable split and stable merger regions.

Finally, the "bigger fish in a smaller pond" logic also generates the counterintuitive result that, for some conditions, the stable-merger region shrinks as α increases:

Proposition 3. *Suppose that $k^i > k^j$ and r is sufficiently large. Then, the stable merger equilibrium is harder to sustain as α increases, and the cycle equilibrium is easier to maintain.*

Recall that α captures the efficiency premium the factions obtain from running together (e.g., the disproportionality of the electoral system). Therefore, it is surprising that increasing α may induce a split in the first period. However, as discussed above, under $k^i > k^j$, faction i has incentives to initiate a cycle and pay a cost today to grab a larger piece of pie tomorrow. The larger α, the larger the pie, the stronger the incentives underlying this dynamics. Thus, increasing α will sometimes increase the parameter region sustaining a cycle equilibrium, instead eroding the stable merger region.

7. Robustness and Extensions

In this section we discuss the results robustness to relaxing some of our assumptions from the baseline model.

7.1. What if both factions can initiate a split

The baseline model assumes for clarity of exposition that only faction i can split. We now examine the robustness of our results to the possibility of both factions initiating a split. Recall that in the original model, faction i has the power to unilaterally initiate a split, while both factions must agree for a re-merger to occur. Consequently, it is clear that the parameter values that generate a stable split or a cycle in the original model also support these equilibria in the expanded version. However one may wonder whether the stable merger equilibrium can be sustained in this enriched setup, or whether it is always the case that if one faction does not want to initiate a split then the other does.

In the Appendix, we analyze this modified version of the model and demonstrate that although the stable-merger region shrinks, there are still parameter values where a stable merger equilibrium exists. The next result shows that when faction j is allowed to initiate a split there is an additional necessary condition to maintain a stable merger equilibrium. Intuitively, in this setting a merger needs to be incentive compatible for both i and j.

Proposition 4. *There exist unique $\underline{k}^i$ and $\underline{k}^j$ s.t. a stable merger equilibrium exists iff $k^i < \underline{k}^i$ and $k^j < \underline{k}^j$.*

Intuitively, for the equilibrium to persist when j can also split, it is crucial that both k^i and k^j are small enough such that neither faction has an incentive to initiate a split and break away from the party.

7.2. Bargaining among factions

We have thus far assumed that the allocation of power and resources within the political party is solely determined by the relative strength of the factions and the party's organizational structure. However, a question arises as to whether faction j can prevent a damaging cycle caused by a split in the first period by offering a more favorable division of the pie to faction i, in exchange for its continued participation in the party.[4] This form of bargaining would be especially beneficial for faction j when $1 > k^i > k^j$, and thus a cycle damages both the party as a whole, and j's relative standing.

In other words, is it possible for the factions to reach a mutually beneficial agreement on the allocation of resources in the first period that would dissuade faction i from splitting? Or instead, even if bargaining is allowed, will cycling still occur in equilibrium?

In this section, it is useful to generalize the assumption on how the strength of the ideological camp a_t evolves over time. In particular, we assume that

$$a_2 = \alpha r^{\gamma}. \tag{7}$$

The parameter $\gamma > 0$ allows ideology to move at a different speed over time (in the baseline, $\gamma = 2$). As in the baseline, $a_1 = \alpha r$.

Suppose then that faction j can offer part of its share of spoils to i if i does not split in $t = 1$. We will focus on the case where j has the strongest incentives to bargain, i.e., $k^i > k^j$. Then we have:

Proposition 5. *Suppose $k^i > k^j$. There exists a $\underline{\gamma}$ such that if $\gamma > \underline{\gamma}$ then a cycle emerges in equilibrium.*

A cycle imposes a large cost on faction j, both today and tomorrow. Thus, we can always find parameter values such that faction j is willing to strike a bargain: offer i part (or all) of its first-period share in order to avoid a split. However, we find that faction i is often unwilling to take the deal. Even if j were to offer the entire pie in the first period, this may not be enough to dissuade i from initiating a damaging cycle.

In particular, when γ is sufficiently high, cycles become inevitable, as factions cannot negotiate beforehand to prevent splits.[5] The party falls victim to a dynamic resource course: The resources available today are insufficient to compensate the splinter faction for the higher gains it

[4] In this section we return to a setup where faction i alone can initiate a split.

[5] Another possible reason for bargaining failure, which we do not consider here, is information asymmetry. If the factions have private information about the possible consequences of a split, the familiar logic from the conflict bargaining literature may lead to an inefficient outcome (see e.g., Fearon 1995).

expects in the future. This highlights a fundamental issue with the allocation of resources within the party: while future party resources are high, factions cannot make binding agreements today on how to split the spoils tomorrow. In fact, in the context of a cycle, any promise made by faction j to offer a larger share of the pie to faction i in the second period is not credible. This is because faction i is willing to re-merge even without this offer, which means that faction j has no incentives to follow through on the promise. This commitment problem leads to an inefficiency, as the party's overall resources are depleted over time.[6]

7.3. Other Considerations

We now consider a few additional potential extensions of the model and discuss how they might affect our qualitative insights.

First, we could allow k^i and k^j to be a function of r, the ideological leaning of the electorate. Intuitively, one may expect the success of the two factions after a split to be related to the ideological strength of the camp as whole. Importantly, this would not fundamentally change our qualitative insights. The model focuses on the net effect of the split, taking into account the ideological leaning of the electorate and all other relevant factors. As long as these factors are captured by the parameter k, the precise functional form of k is less important.

Second, one might think that the effect on a faction's brand generated by a split is not long-lasting if factions re-merge in the same party again. Suppose for example, that the splinter faction presents itself as appealing to a more extreme and 'ideologically pure' portion of the electorate. This allows the faction to cultivate a certain identity, or brand. It seems plausible, then, that some of this identity may be lost should the faction re-merge again with its moderate counterpart. To capture this intuition, we could allow re-merging to dampen some of the effect of splits on the factions' brands. This could be done by introducing a parameter that captures the degree to which a re-merged faction retains the same brand accumulated during the split. Again, this extension would not change our qualitative insights as long as the effect of re-merging is not so strong as to completely erase the gains or losses incurred from the split.

[6]The result is also reminiscent of Powell (2004), who shows how large, rapid changes in the bargainers' relative power cause inefficiency, even with complete information.

Finally, we could allow brands to evolve somewhat during merger periods. That is, factions could still develop their own ideological identity, or organizational capacity, thus differentiating from each other while in the same party. This is especially true in parties that allow for internal competition, for example through primaries. Similarly to the previous case, we could introduce a parameter that captures the extent to which factions' brands evolve when factions are within the party. As long as the evolution is muted compared to after a split, our main results would still hold.

8. Conclusion

Most party systems frequently witness significant political changes, with splits and mergers of political parties taking center stage. This has led to a growing interest among scholars and political observers in understanding the complex dynamics of party politics and factionalism. This paper develops a theory to explain why factions belonging to the same party might choose to split, and when instead we should expect party unity.

Our model produces some intuitive results. On the one hand, when a faction benefits a lot from splitting (e.g., because the new faction leadership gains visibility or voters perceive the new party as ideologically pure), in equilibrium party unity is not sustainable, and different factions diverge on their own separate paths. On the other hand, when the added benefit of running together is very high (perhaps because electoral institutions are very disproportional), factions do not split in equilibrium, thus providing a possible explanation for party system stability.

Other results generated by our model are more surprising. We show that factions may split from their party even when, by doing so, they damage both themselves and their ideological camp. These damaging splits can only be sustained in equilibrium as part of a 'cycle', whereby factions split today only to re-merge tomorrow. One dynamic that might sustain such a cycle is when a split damages both factions, and the splinter faction anticipates to improve its relative standing within the party, thus becoming a bigger fish in a smaller pond. A second, perhaps less intuitive dynamic occurs when a split damages the splinter faction but advantages the remaining one, and the splinter agrees to split to enjoy a smaller share of bigger pie tomorrow, thus becoming a smaller fish in a bigger pond. Importantly, these cycles do not occur because of changes in the underlying conditions that alter what is statically optimal for factions. In fact, dynamic incentives can push factions to split (and then re-merge), even if splitting is statically damaging. These different dynamics generate rich comparative statics, whereby forces that would intuitively push towards party stability may actually incentivize splits in equilibrium in our model.

Overall, our model provides insights into the workings of political parties and the role of factionalism in party development. Despite growing recognition of the internal divisions that can characterize political parties, the intra-party dynamics driving these fissures are too often overlooked as a key driver of party system change. In turn, our approach produces implications that can be applied to several political systems. We believe that our model can provide a foundation for further analysis, informing and stimulating future research into the importance of factional politics and the various outcomes it can produce.

While our results would persist in a world with limited uncertainty about the consequences of splits, a natural question to ask is how facing substantive uncertainty would change factions' incentives to split, and whether the equilibria we uncover persist in a high-uncertainty world. For example, in such a setting splits might happen for experimentation: while factions have a perception of what could happen if they split when they belong to the same party, it is only when they split that these perceptions become unequivocal signals. We believe that analyzing factions' incentives to experiment in this more complex setup is perhaps the most promising avenue for future theoretical research departing from our model. This would allow to make predictions on party evolution as a consequence of factions' experimentation.

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Appendix

Proof of Proposition 1. To prove this proposition, we will proceed as follows. First, we will show that there exists no equilibrium where the factions remain merged in the first period and split in the second. Next, we identify conditions for existence of the three equilibria characterized in the Proposition. Finally, we will establish equilibrium existence and uniqueness.

Conjecture an equilibrium where the factions remain merged in the first period and split in the second. First, consider faction i's choice to split in $t = 2$ given that factions were merged in $t = 1$. This is optimal if and only if

$$b_0 k^i > b_0 + \frac{1}{2}\alpha r^2, \tag{8}$$

which requires $k^i > 1$.

To evaluate factions' behavior in the first period, we need to consider the possible off-path conditions. First, suppose that the equilibrium prescribes factions to stay split after a deviation in $t = 1$. In this case, the equilibrium requires:

$$b_0 + \frac{1}{2}\alpha r > b_0 (k^i)^2, \tag{9}$$

which contradicts condition 8.

Next, suppose that the equilibrium prescribes factions to merge after a deviation in $t = 1$, that is:

$$b_0 (k^i)^2 < \left(b_0(k^i + k^j) + \alpha r^2\right)\left(\frac{1}{2} + \phi b_0 (k^i - k^j)\right). \tag{10}$$

Then, the equilibrium requires that in $t = 1$:

$$b_0 + \frac{\alpha r}{2} > \left(b_0(k^i + k^j) + \alpha r^2\right)\left(\frac{1}{2} + \phi b_0 (k^i - k^j)\right), \tag{11}$$

The last two conditions imply

$$b_0 (k^i)^2 < b_0 + \frac{\alpha r}{2}, \tag{12}$$

which contradicts the second period equilibrium condition (8).

We will now move to identifying conditions for the existence of the three equilibria characterized in the Proposition.

Stable splits.

By backward induction, consider faction i's choice to split in $t = 2$, given that factions split in $t = 1$. Faction i splits in equilibrium if and only if:

$$b_0(k^i)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right]. \tag{13}$$

Which rearranges to $b_0(k^i)^2 - \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > 0$. Since $\frac{1}{2} + \phi b_0(k^i - k^j)$ is always strictly positive, the condition is never satisfied at $k^i = 0$. Further, differentiating the LHS with respect to k^i we obtain

$$b_0(-\frac{1}{2} + (2 - 2b_0\phi)k^i - \alpha\phi r^2),$$

which is convex in k^i. This follows from the assumption that $\phi < \frac{1}{2b_0\bar{k}^2}$. The condition is never satisfied at $k^i = 0$, therefore it must establish a lower bound $\underline{k}^i$, such that faction i splits for $k^i > \underline{k}^i$. It is easy to see that $\underline{k}^i < \bar{k}$ for a sufficiently large $\bar{k}$ (i.e., $\underline{k}^i$ is finite).

Alternatively, it could be that factions split in $t = 2$ if i prefers to merge (Equation 13 does not hold) but j does not:

$$b_0(k^j)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^j - k^i)\right]. \tag{14}$$

Notice that, as for 13, condition 14 establishes a lower bound such that factions split for $k^j > \underline{k}^j$.

Consider now faction i's choice to split in $t = 1$. On the equilibrium path, this yields an overall payoff of:

$$b_0k^i + b_0(k^i)^2. \tag{15}$$

First, suppose that the equilibrium prescribes factions to stay merged after a deviation in $t = 1$. This off-path behavior requires no incentive to split, that is:

$$\frac{1}{2}\left[2b_0 + \alpha r^2\right] > b_0k^i. \tag{16}$$

Given Equation 16, faction i splits in $t = 1$ in equilibrium if and only if:

$$b_0 k^i + b_0 (k^i)^2 > 2b_0 + \frac{1}{2}\alpha r(1 + r), \tag{17}$$

Alternatively, suppose that the equilibrium prescribes faction i to split after a deviation in $t = 1$ (i.e., condition 16 does not hold). Then, faction i splits in $t = 1$ if:

$$b_0 k^i + b_0 (k^i)^2 > \frac{1}{2}\left(2b_0 + \alpha r\right) + b_0 k^i. \tag{18}$$

Both conditions 17 and 18 establish lower bounds on k^i, and one or the other will be binding depending on whether 16 holds or not. We denote the binding lower bound $\underline{k}^i_s$.

Stable mergers.

By backward induction, consider faction i's choice to stay merged in $t = 2$, given that factions were merged in $t = 1$. This choice is optimal if and only if

$$b_0 + \frac{1}{2}\left(\alpha r^2\right) > b_0 k^i. \tag{19}$$

To evaluate factions' behavior in the first period, we need to consider the possible off-path conditions. First, suppose that the equilibrium prescribes factions to merge after a deviation in $t = 1$. That is, either i wants to remain split in $t = 2$:

$$b_0 (k^i)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0 (k^i - k^j)\right], \tag{20}$$

or i wants to merge but j does not: i.e., Equation 35 does not hold and

$$b_0 (k^j)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0 (k^j - k^i)\right]. \tag{21}$$

Thus, in the first period, faction i does not split if and only if:

$$\frac{1}{2}(2b_0 + \alpha r) + \frac{1}{2}(2b_0 + \alpha r^2) > b_0 k^i + b_0 (k^i)^2, \tag{22}$$

which establishes an upper bound on k^i.

Suppose instead that factions re-merge after a deviation, which requires:

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > b_0(k^i)^2, \tag{23}$$

and

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^j - k^i)\right] > b_0(k^j)^2. \tag{24}$$

In this case, in $t = 1$, faction i does not split if and only if:

$$\frac{1}{2}(2b_0 + \alpha r) + \frac{1}{2}(2b_0 + \alpha r^2) > b_0 k^i + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right]. \tag{25}$$

Which again establishes an upper bound on k^i. Thus, depending on the parameter values either 19, 22 or 25 will be binding, and there exists a unique $\widehat{k}^i$ s.t. a stable merger equilibrium exists if and only if $k^i < \widehat{k}^i$.

Cycles.

Consider factions' choice to merge in $t = 2$, given that there is a split in $t = 1$. Both factions need to profit from merging, i.e.,

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > b_0(k^i)^2 \tag{26}$$

and

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^j - k^i)\right] > b_0(k^j)^2, \tag{27}$$

which establish upper bounds on both k^i and k^j.

Consider now the first period behavior. First, suppose that the equilibrium prescribes factions to stay merged after a deviation, i.e. condition 16 holds. This implies that faction i splits in equilibrium if and only if:

$$b_0 k^i + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > 2b_0 + \frac{1}{2}\left(\alpha r + \alpha r^2\right), \tag{28}$$

which requires that k^i is high enough.

Alternatively, suppose that the equilibrium prescribes factions to split after a deviation in $t = 1$ (i.e., condition 16 does not hold). This implies that faction i splits in equilibrium if and only if:

$$b_0 k^i + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > 2b_0 + \frac{1}{2}(\alpha r) + b_0 k^i, \tag{29}$$

which rearranges to

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] - 2b_0 - \frac{1}{2}(\alpha r) > 0. \tag{30}$$

By inspection, we can see that the LHS is convex in k^i.

Together with Equation 26 then, 28 and 29 imply that there exist $\overline{k}^j$, $\underline{k}^i$ and $\overline{k}^i$ s.t. a cycle equilibrium exists iff $k^i \in [\underline{k}^i, \overline{k}^i]$ and $k^j < \overline{k}^j$.

Finally, we establish that a pure strategy equilibrium always exists, and that the equilibrium is unique, except for a measure-zero set of parameter values.

First, notice that we cannot sustain mixed strategy in the first period with pure strategies in the second. Next, it is easy to see that only a measure-zero set of parameters can sustain indifference in the second period. Thus, except for this measure-zero set, the equilibrium must be in pure strategy, and (under $r > 1$) the equilibrium must be one of the three characterized above.

We now establish uniqueness. First, notice that there exist no parameter values for which both a stable split and a cycle equilibrium exist, since conditions 26 and 27 are the complements of conditions 13 and 14, respectively.

Next, notice that there exist no parameter values for which both a stable merger and a stable splits equilibrium exist. Suppose that a stable splits equilibrium exists. Then, a possible stable merger equilibrium may only be sustained with a strategy prescribing that i splits and factions remain split off the equilibrium path and conditions $b_0 + \frac{1}{2}\alpha r^2 > b_0 k^i$ and $2b_0 + \frac{1}{2}\alpha r(1 + r) > b_0 k^i(1 + k^i)$. In turn, if $b_0 + \frac{1}{2}\alpha r^2 > b_0 k^i$ then the stable split equilibrium can only be sustained by a strategy prescribing that factions never split off the equilibrium path. However, this would require $2b_0 + \frac{1}{2}\alpha r(1 + r) < b_0 k^i(1 + k^i)$, which contradicts the previous condition.

Finally, there exists no parameter value for which both a stable merger and a cycle equilibrium exist. Suppose that a cycle equilibrium exists. Then, a possible stable merger equilibrium may only be sustained with a strategy prescribing that i splits and factions re-merge off the equilibrium path and conditions $b_0 + \frac{1}{2}\alpha r^2 > b_0 k^i$ and $2b_0 + \frac{1}{2}\alpha r(1+r) > b_0 k^i + (b_0(k^i + k^j) + \alpha r^2)(\frac{1}{2} + \phi b_0(k^i - k^j))$. In turn, if $b_0 + \frac{1}{2}\alpha r^2 > b_0 k^i$ then the cycle equilibrium can only be sustained by a merger-merger off-path. However, this would require $2b_0 + \frac{1}{2}\alpha r(1+r) < b_0 k^i + (b_0(k^i + k^j) + \alpha r^2)(\frac{1}{2} + \phi b_0(k^i - k^j))$.

Therefore, there can be no overlap in the regions sustaining different equilibria, i.e., it must be the case that $\widehat{k^i} = \underline{k}^i \leq \overline{k}^i = \widetilde{k}^i$. □

Proof of Corollary 1. Follows from the conditions identified above. □

Proof of Proposition 2. Follows from inspection of the existence conditions. □

Proof of Proposition 3. Suppose that r is sufficiently large that 19, 23 and 24 hold. Then, in equilibrium we have a stable merger if

$$b_0 k^i + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] - 2b_0 - \frac{1}{2}\left(\alpha r + \alpha r^2\right) < 0, \tag{31}$$

and a cycle if

$$b_0 k^i + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] - 2b_0 - \frac{1}{2}\left(\alpha r + \alpha r^2\right) > 0. \tag{32}$$

Differentiating the LHS wrt to α we get

$$r\phi b_0(k^i - k^j) - \frac{1}{2}, \tag{33}$$

which is positive for a sufficiently large r. □

Proof of Proposition 4. Intuitively, if we consider parameter values for which we would have a stable split in equilibrium in the baseline, we continue to have a stable split in this case. Similarly, because a re-merger always requires j's consent, under the conditions sustaining a cycle in the baseline we continue to have a cycle here. Thus, here we will focus on the case in which $k^i < \underline{k}^i$, i.e., in the baseline we have a stable merger in equilibrium.

Here, we will show that while allowing j to split erodes the stable-merger region, this kind of equilibrium continues to arise for some parameter values.

We proceed exactly as in the proof for the baseline case.

By backward induction, consider faction j's choice to stay merged in $t = 2$, given that factions were merged in $t = 1$. This choice is optimal iff

$$b_0 + \frac{1}{2}\left(\alpha r^2\right) > b_0 k^j. \tag{34}$$

To evaluate factions' behavior in the first period, we need to consider the possible off-path conditions. First, suppose that the equilibrium prescribes factions to remain split after a deviation in $t = 1$. That is, either i wants to remain split in $t = 2$:

$$b_0(k^i)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right], \tag{35}$$

or i wants to merge but j does not: i.e., Equation 35 does not hold and

$$b_0(k^j)^2 > \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^j - k^i)\right]. \tag{36}$$

Thus, in the first period, faction j does not split if and only if:

$$\frac{1}{2}(2b_0 + \alpha r) + \frac{1}{2}(2b_0 + \alpha r^2) > b_0 k^j + b_0(k^j)^2, \tag{37}$$

which establishes an upper bound on k^j.

Suppose instead that factions re-merge after a deviation, which requires:

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^i - k^j)\right] > b_0(k^i)^2, \tag{38}$$

and

$$\left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} + \phi b_0(k^j - k^i)\right] > b_0(k^j)^2. \tag{39}$$

In this case, in $t = 1$, faction j does not split if and only if:

$$\frac{1}{2}(2b_0 + \alpha r) + \frac{1}{2}(2b_0 + \alpha r^2) > b_0 k^j + \left[b_0(k^i + k^j) + \alpha r^2\right]\left[\frac{1}{2} - \phi b_0(k^i - k^j)\right]. \tag{40}$$

Which again establishes an upper bound on k^j. Thus, depending on the parameter values either 34, 37 or 40 will be binding, and there exists a unique $\underline{k}^j$ s.t. a stable merger equilibrium exists if and only if $k^j < \underline{k}^j$. □

Proof of Proposition 5. Suppose that the conditions for a cycling equilibrium hold, i.e.: Equations 26, 27, 28 and 16 all hold. An interesting question is whether factions can agree on a division of surplus in $t = 1$ that prevents faction i from splitting in $t = 1$, or whether cycles still occur in equilibrium even if allowing for bargaining.

Before proceeding with the analysis, let us generalize the assumption on how a_t evolves over time. In particular, we assume that

$$a_2 = \alpha r^\gamma. \tag{41}$$

The parameter $\gamma > 0$ allows ideology to move at a different speed over time (in the baseline, $\gamma = 1$). As in the baseline, $a_1 = \alpha r$.

Suppose that faction j can offer part of its share of spoils to i if i does not split in $t = 1$. We will focus on the case in which j has the strongest incentives to bargain, i.e., $k^i > k^j$.

In equilibrium, j will offer either 0 or the x such that i is indifferent between staying in the party and splitting:

$$x + \frac{2b_0 + \alpha r}{2} + \frac{2b_0 + \alpha r^\gamma}{2} = b_0 k^i + \left(b_0(k^i + k^j) + \alpha r^\gamma\right)\left(\frac{1}{2} + b_0\phi(k^i - k^j)\right), \tag{42}$$

which yields the following solution:

$$x^b = \frac{2b_0^2\phi\left((k^i)^2 - (k^j)^2\right) + b_0\left(k^i\left(2\alpha\phi r^\gamma + 3\right) - 2\alpha k^j\phi r^\gamma + k^j - 4\right) - \alpha r}{2} \tag{43}$$

Faction j offers x^b in equilibrium if two conditions hold. First, faction j has to prefer to give away x^b today to the payoff from the cycle equilibrium. Formally:

$$\frac{1}{2}(2b_0 + \alpha r) - x^b + \frac{1}{2}\left(2b_0 + \alpha r^{\gamma}\right) > b_0 k^j + \left[b_0(k^i + k^j) + \alpha r^{\gamma}\right]\left[\frac{1}{2} + b_0 \phi(k^j - k^i)\right]. \tag{44}$$

Substituting the value of x^b, the above simplifies to

$$\alpha r > 2b_0(k^i + k^j - 2). \tag{45}$$

Equation 45 shows that faction j is willing to give up part of its share as long as the value of staying together (the LHS) is high enough.

Second, x^b has to be feasible, i.e., it cannot exceed j's share of the pie. That is, $x^b \in \left[0, \frac{2b_0+\alpha r}{2}\right]$. Substituting the value of x^b we obtain the following condition:

$$\alpha r > b_0^2 \phi \left((k^i)^2 - (k^j)^2\right) + \frac{1}{2} b_0 \left[k^i\left(2\alpha r^{\gamma}\phi + 3\right) + k^j\left(1 - 2\alpha r^{\gamma}\phi\right) - 6\right]. \tag{46}$$

Suppose $k^i > k^j$ and consider Equation 46: as $\gamma \to \infty$, the RHS clearly tends to infinity as well. Thus, there must be a value $\hat{\gamma}$ such that for $\gamma \leq \hat{\gamma}$ the budget constraint condition 46 holds, while for $\gamma > \hat{\gamma}$ it does not. Further, it is easy to see that the conditions for the existence of a cycling equilibrium in the no-bargaining subgame can always be sustained under a sufficiently high γ. Thus, there exists a threshold $\underline{\gamma}$ for which a cycle emerges in equilibrium even if we allow for bargaining in the first period. $\square$